ATP - The Airsoft Trajectory Project

The ATP is undergoing a major rework. As of such, some links many not work. If you come across incomplete or misdirecting links, please notify me and I will fix them as soon as possible. Thank you.

Introduction

I'm always trying to figure out the science behind the sport. Whether it's golf, football, tennis, or airsoft, I like to get into the details of what happens in terms of physics. Taking this approach to airsoft, I decided to set out to estimate trajectories based on experiments and the myriad equations for calculating trajectory. In the end, I had compiled the equations necessary to calculate the many forces acting upon airsoft BB's, coupled that with a little know-how and developed a program in MATLAB to ultimately calculate projectile trajectory. Basically, the program can be used for paintball, regular 0.177 BB's, as well as airsoft BB's. It is designed to account for:

- **muzzle velocity**
- **mass of projectile**
- **diameter of projectile**
- **altitude**
- **temperature**
- **air density / pressure**
- **wind and wind direction**
- **crosswind component**
- **amount of hop-up applied**

The gun the BB is fired from is irrelevant. Once the BB leaves the barrel, it has no memory of the gun it was fired from. It has a magnitude (several, technically) and a vector, and in terms of physics, that's all that matters. Consequently, I needed to model the data knowing the initial velocity of the BB, the direction of the BB, and the spin that the BB incurs from hop-up. Granted, different guns will have a slightly different directional component when the BB exits the barrel (and we're talking VERY SLIGHT), and hop-up varies from unit to unit, however the program assumes that the BB is following the path dictated by the direction of the barrel, that the muzzle velocity is, at worst, +/- 2% of it's average muzzle velocity, and that the hop-up is capable of putting a consistent amount of backspin on the BB (i.e., it has been "broken in").

Before publishing calculated results, I went to great lengths to make sure that the calculations were accurate in terms of describing the actual trajectory. I spent several months performing tests, gathering data, contacting other scientists in the know, reading a plethora of theses, collecting information from others who had collected data, staying up late at night jotting down equations (and scratching through yet more equations), testing algorithms, performing tests AGAIN to ensure that I hadn't made errors in my methods, and of course many hours of the standard vitriolic spewing that occurs when you just can't get the programs to work.

Ultimately, all of the testing verified the final calculations. For more information on testing methods and validation, consult Section II: Testing and Model Validation.

Having verified the data, the next thing I wanted to do was calculate standard trajectory, hop-up trajectory, energy dissipation, velocity reduction, time of flight, minimum engagement distances, and the effects of altitude and temperature for a wide variety of muzzle velocities and BB weights and post the results online. Hopefully it will answer many of the questions people are putting forth about airsoft rifles, questions such as:

- What is the terminal velocity of a 0.20g BB?

- Is it worth upgrading a gun from x fps to y fps to get more range?

- For equal muzzle energies, which BB goes further, 0.20g or 0.25g?

- Which mass BB gets to the target the quickest for the given velocity?

- Do heavier mass BB's have more energy than lighter one down range?

- Is it necessary to restrict a rifle with a 600 fps muzzle velocity to a minimum engagement

distance of 100 feet?

- What MED's are recommended to ensure both safety and fairness to all shooters?

- Will lower temperatures increase or decrease range?

- Does altitude really affect trajectory and minimum engagement distances?

- Do 0.43 gram BB's negate the effects of wind *that* much better than 0.20 gram BB's?

- Do 8mm BB's resist the effects wind better than 6mm BB's?

- Do high-velocity BB's resist wind better than low-velocity BB's?

- What's the effective range on my rifle?

- What's the absolute maximum range on my rifle?

- Are people really able to achieve ranges out to 300 feet?

- Do 8mm BB's provide better range than 6mm BB's?

Those are all good questions. Unfortunately, I've seen many answers out there that are, at best, just guesses. And more often than not, I've seen answers that simply disagree with the laws of physics. Hopefully all of the data will provide people with some answers to these and other questions. If you find yourself in disbelief over what the calculations depict, spend some time looking at the equations; they're the standard equations used for this sort of thing and are universal when it comes to ballistics. Even if you're still not convinced, spend some additional time reading about the methodology used to verify the equations. If you're not convinced after that, I encourage you to do some testing and see how your results compare.

Additionally, I realize that airsoft is an inexact science. Air pockets, surface bumps, diameter inconsistencies, shifting winds, muzzle velocities inconsistencies... these things and others lead to erratic behavior in a BB's trajectory down range. Even so, I think that it is better to have a rough idea of what the "ideal BB" would do in flight, and allow the shooter to factor in their own "fudge factor."

In terms of usefulness, those using upgraded guns or guns that tend to have a high degree of accuracy will benefit most from the data. If your gun's muzzle velocity varies by 20-30 fps per shot or if, for a variety of reasons, your gun is incapable of reproducing the same trajectory shot after shot (and frankly some of mine fit that description) then the data may be less useful. Ultimately the usefulness of the data will be determined by the end user and will still be dependent upon how familiar the user is with their gun. For me, I think that it is very handy to have. But of course, I am biased.

Anyway, here are the data. It's divided up into many sections as it's all a little overwhelming (with around 270 charts and graphs). It's designed to be read from start to finish so if you can't figure out something in a later section, chances are that the explanation was provided in an earlier section. (And, if you're still trying to make sense of all of the questions above, they're answered concisely in **Section VIII: Closing Remarks**.)

If you have a question or comments about the data, or would like to see some additional analysis or data plots, feel free to contact me. Particularly if you want advice or graphics depicting recommended Minimum Engagement Distances, drop me a line and I'll try to help make some tailor-made plots for use at your airsoft site.

Lastly, while I consider the hop-up trajectories to be close, they're not perfect; if you have an opinion or have observational information concerning trajectory, by all means drop me a line. Doing so will help to modify some of the coefficients that affect hop-up calculations. I hope to eventually post a program online for people to download so that they can calculate trajectory for their own rifles, however given the lack of copious spare time, it may be another year or so before I can develop the online calculator.

Nathan

December, 2006

I. Physics Principles and Equations

A. MilDot Scope Calculator

B. Relative Energy / Minimum Engagement Distance Calculator

The Airsoft Trajectory Project

ATP - Physical Characteristics of BB's

web.archive.org/web/20200620060317/http://mackila.com/airsoft/atp/01-a-01.htm

The average diameter of a BB tends to be a little less than 6mm but for all practical purposes we'll use a diameter of 6mm. I did separate calculations using a diameter of 5.90 mm and determined that the results were so similar that it wasn't worth modifying -- I kept it at 6mm.

For comparison's sake, I've included the diameter's of standard BB's as well as paintballs:

Type	Diameter (mm)	Caliber	Frontal Area (mm^2)	Frontal Area (m^2)
Airsoft	6	0.235	28.27	0.0000283
Airsoft - Large	8	0.315	50.27	0.0000503
Steel BB's	4.5	0.177	15.90	0.0000159
Paintball	17.2	0.68	235.06	0.0002351

Type	Diameter (mm)	Mass (grams)	Mass (grains)	Volume (m^3)	Density (kg/m^3)	Terminal Velocity	
						(fps)	(mph)
Airsoft	6	0.12	1.9	0.0000001131	1061	40.0	27.3
Airsoft	6	0.20	3.1	0.0000001131	1769	51.6	35.2
Airsoft	6	0.25	3.9	0.0000001131	2211	57.7	39.3
Airsoft	6	0.30	4.6	0.0000001131	2653	63.2	43.1
Airsoft	6	0.36	5.6	0.0000001131	3184	69.2	47.2
Airsoft	6	0.43	6.6	0.0000001131	3803	75.7	51.6
Airsoft - Large	8	0.34	5.2	0.0000002680	1269	50.1	34.1

Airsoft - Large	8	0.45	6.9	0.0000002680	1679	57.6	39.3
Daisy BB	4.5	0.33	5.1	0.0000000477	6916	87.7	59.8
Copperhead BB	4.5	0.36	5.5	0.0000000477	7545	91.6	62.4
Dynamic BB	4.5	0.42	6.5	0.0000000477	8803	98.3	67.4
Daisy Heavy BB	4.5	0.45	6.9	0.0000000477	10689	102.4	69.8
*Copperhead Pellet	4.5	0.51	7.9	--	--	--	--
Paintball	17.2	3.2		0.0000026972	1201	71.4	48.7
* Pellets range from about 5 to 9 grains; 7.9 is by no means the weight of all pellets. Additionally, because of the non-uniform shape of pellets, it's impossible to accurately calculate volume and density (without using AutoCAD).							

Notice that 8mm BB's, though heavier, are still less dense as compared to 0.20g 6mm BB's.

To give you an idea of other densities, at sea level and 15 C, air density is 1.225 $^{kg}/_{m3}$, which is nearly a thousand times less dense than even 0.12g BB's. Just for the sake of comparison, here are some other densities: fresh water (1000 $^{kg}/_{m3}$) and salt water (1027 $^{kg}/_{m3}$), which is why even 0.12g BB'swill sink in water. As an additional comparison, the density of different 0.177” bb’s ranges from 6916 to 10689 $^{kg}/_{m3}$ whereas a standard paintball has a density of 1201 $^{kg}/_{m3}$.

It is interesting to note that the terminal velocity of 0.20g 6mm BB's is slightly over 50 fps. I have read where people state that an airsoft pellet, when fired straight upward, will land with the same velocity that it was fired at. This is erroneous thinking. After reaching the apex in its trajectory, a BB will accelerated downward until it reaches its terminal velocity. This phenomena is best illustrated in paintball. A paintball may be fired skyward at over 300 fps, but lands it such a low velocity that it often bounces off of the ground as opposed to shattering. The reason for this is that the paintball -- fired upward at 300 fps -- falls at its terminal velocity, or roughly 70 fps.

(Just for greater explanation, terminal velocity is NOT the maximum speed of a projectile, but rather the velocity of a falling object wherein the force of gravity is negated by the force of drag. Once an object in free fall reaches its terminal velocity, it will not fall any faster.)

Air Density

> $\mathbf{p = p_0 * \{ 1 + [(L * h) / T_0] \}^{[(g * M) / (R * -L)]}}$
>
> where **g** is the average gravitational acceleration at the Earth's surface of 9.80655 $^{m} / _{s}2$
>
> and $\mathbf{p_0}$ is standard pressure at sea level, or 101,325 $^{kg} / _{m * s}2$

> $\mathbf{T = T_0 + L * h}$
>
> where $\mathbf{T_0}$ is the standard temperature at sea level, or 288.15 K (converting 15 C to Kelvin)
>
> where **L** is the adiabatic lapse rate for dry air of -0.0065 $^{K} / _{m}$
>
> and **h** is the altitude in meters above sea level

As temperatures increase or as altitude increases, air density decreases. A ballpark rule of thumb is that for a 10 F (or roughly 5.5 C) increase in temperature, air density will decrease by 0.02 $^{kg} / _{m}3$. Also, as altitude increases by 500 ft (or 328m), air density will again decrease by 0.02 $^{kg} / _{m}3$. (And again, these are ballpark figures, for an example.)

Note that my calculations do not take into account the amount of water vapor in the air (relative humidity). This was done initially however the effects of humidity on density were so negligible that it was removed from the equation. Even temperature and altitude -- both of which have a much more pronounced effect on air density than humidity -- still account for very little variation in performance (though it is still observable and thus I have included charts detailing the effects of altitude and temperature on trajectory).

As humidity increases, it causes the air to become less dense. This is certainly counter-intuitive. One would think that adding water vapor would add to the air density simply because water is much more dense in comparison to air. But we're not adding *water*; we're adding *water vapor*. Water vapor is less dense than air. The reason for this is the molecular weight of water vapor is lower than that of standard air. Think of it this way: air is, for all practical purposes, primarily comprised of nitrogen and oxygen with nitrogen constituting 78% and oxygen 21% of total air volume (with Argon and CO_2 making up almost all of the remaining 1%). The molecular weight of oxygen and nitrogen combined is, simply, heavier than that of the combination of hydrogen and oxygen. Therefore, adding water vapor to air causes the combined medium to become a less dense than dry air.

The amount of water vapor that can be held in air is related to the temperature of the air. Simply put, warmer air can hold more water vapor. If you compared the density of air that is 40 F with 0% relative humidity to that of 40 F air with 100% humidity, you'd find that the density difference is very, very small. Even at higher temperatures (and again, warmer air can hold more water vapor, affecting density to a greater degree) it's still a negligible factor. The change in density of air that is 90 F with 0% relative humidity to that of 90 F with 100% humidity is about 0.002 $^{kg}/_{m^3}$. Taking humidity into account for calculating air density wouldn't give us any benefit since the subtle change of 1 degree Fahrenheit will have more of an effect on the air density. And in reality, a BB's trajectory will more than likely take it through several regions of air sporting different temperatures (as in passing over a patch of sand, or a patch of grass, over asphalt, through a region of air that is in the shade, etc.).

ATP - Kinetic Energy

Section I-C: Kinetic Energy

Calculating the kinetic energy of a projectile is fairly straight forward using the following equation:

> $KE = {}^1/_2\ m * v^2$

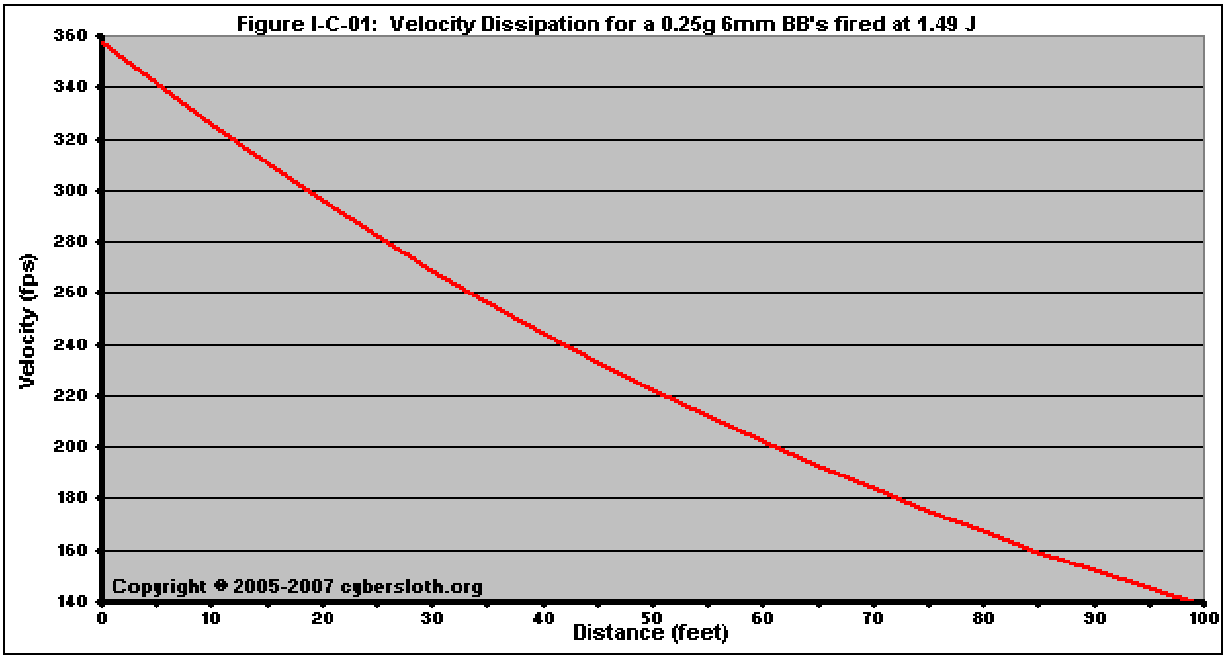
where **m** is the mass of the projectile in kilograms, and **v** is the velocity in meters per second.

In Airsoft, energy is unimportant in calculating trajectory, however it is very important when determining safe minimum engagement distances (or MED's). In terminal ballistics, the impact energy in relation to impact area is ultimately what determines whether or not a projectile will penetrate a given target. In the UK, for instance, they've determined that an impact greater than 1.35 Joules (with 6mm Projectiles) is capable of serious harm (though I have been unable to find further specifics on this). To further ensure safety, the recommended minimum engagement distances in this study (see Section VI-C: Recommended Universal MED's) allow for a maximum impact energy no greater than 1.00 Joule.

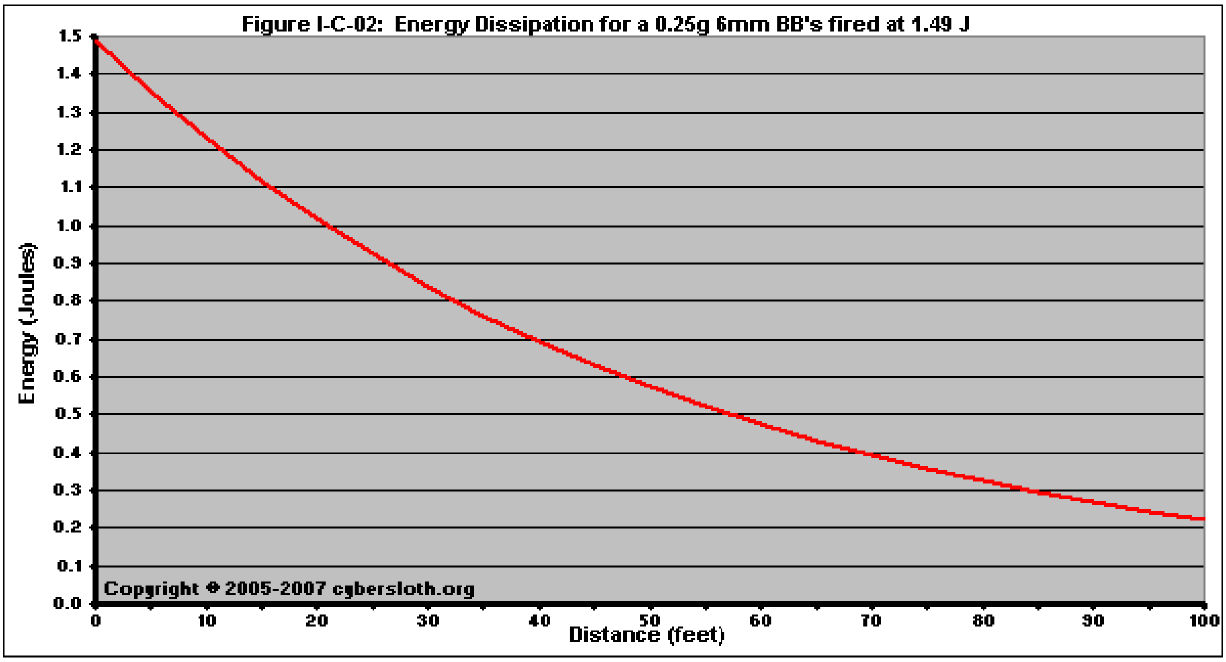
For example, a 0.25g 6mm BB fired at 358 fps would have a muzzle energy of 1.49 Joules, calculated as follows:

> $KE = {}^1/_2\ m * v^2 = 0.5 * 0.00025\ kg * 109\ m/s^2 = 1.49\ kg * m^2/s^2 = 1.49\ Joules$

As the BB encounters drag throughout its trajectory, its velocity diminishes, as shown in Figure I-C-01:



As velocity decreases, so to does the kinetic energy. In order for a BB to have a safe impact, its velocity will need to decrease to the point where its kinetic energy is less than 1.00 Joules. A 0.25g BB will have 1.00 J of energy at 293 fps. From the above chart, we can see that the BB's velocity will reach 293 fps at a range of roughly 21 feet. This is further illustrated in Figure I-C-02, wherein we can see that, at a distance of about 21 feet, the 0.25g BB has reached the 1.00 J mark:

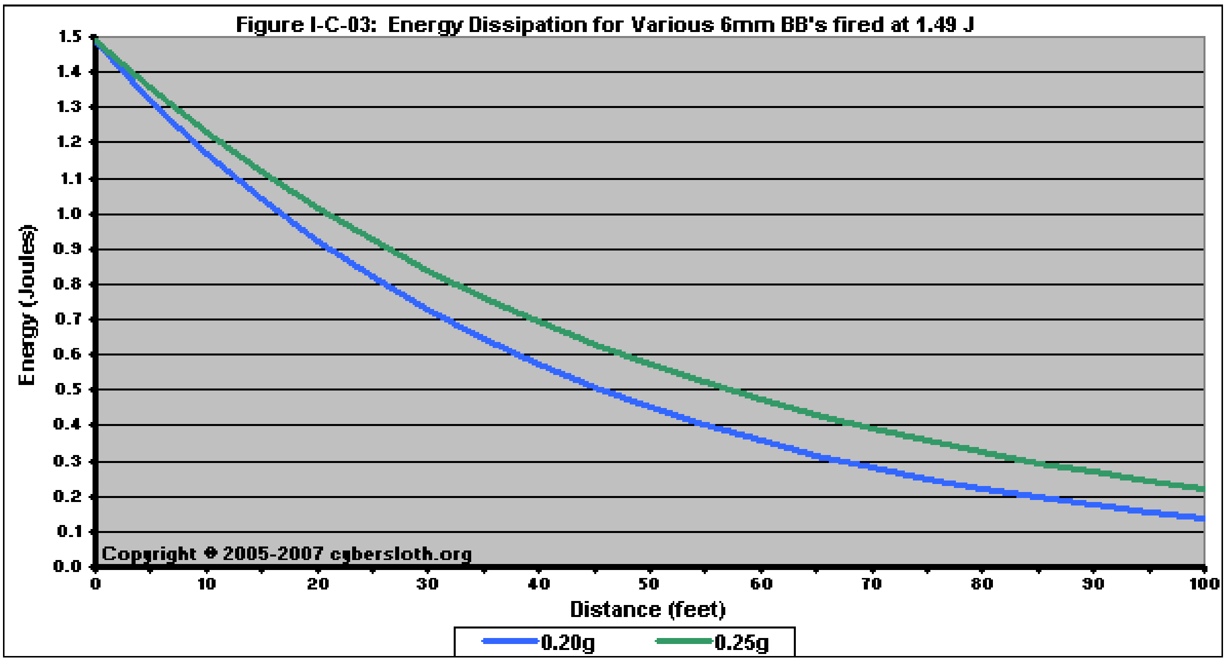


Most rifles below 1.5 J are fairly uniform in terms of muzzle energy. In other words, a stock rifle that fires a 0.20g BB at 0.75 J will probably fire a 0.25g BB at 0.75 J. To keep the energy the same, a heavier BB will need to be moving slower. A rifle with a muzzle energy of 0.75 J would fire 0.20g BB's at 285 fps, and it would fire 0.25 g BB's at 255 fps. (To calculate velocities for an equal energy, see Section X-B: Relative Energy / MED Calculator.)

For rifles above 1.5 J, muzzle energy is not always constant for different masses. As is explained in Section VI-A: Determining Muzzle Energy, lighter BB's in high-energy rifles are -- particularly gas rifles -- sometimes exit the barrel before they have had a chance absorb all of the energy from the spring compression or gas expansion. This is not true for all high-energy rifles, but occurs often enough to be worth noting.

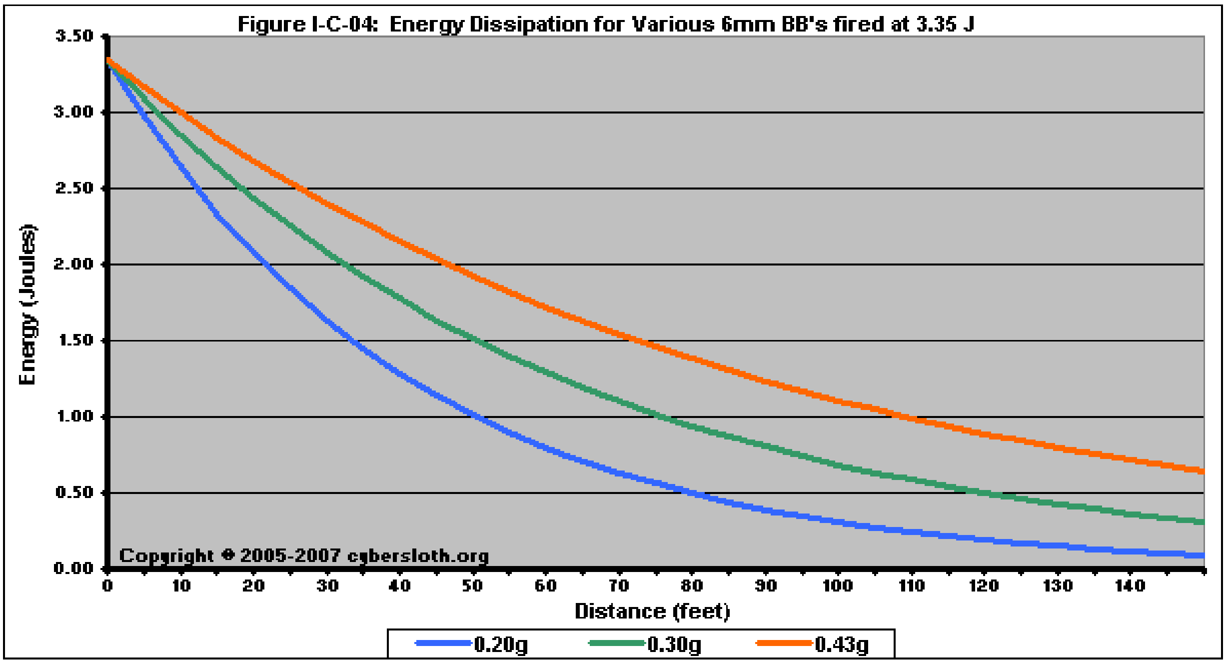
One of the more interesting things, something of particular importance when determining MED's, is that for equal muzzle energies, heavier BB's dissipate energy at a slower rate. There are two reasons for this. First, for equal muzzle energies, a lighter BB will be moving faster than a heavier one. As you will see in Section I-D-01: Drag Force, drag increases with the square of the velocity. A BB moving at 400 fps, for example, will experience considerably more drag than a BB moving at 300 fps. The second reason that heavier BB's dissipate energy at a slower rate is that the deceleration a BB experiences is inversely proportional to the mass of the BB. As the mass increases, the rate of deceleration decreases.

The sum of these reasons can be seen in the Energy Dissipation plot in Figure I-C-03:



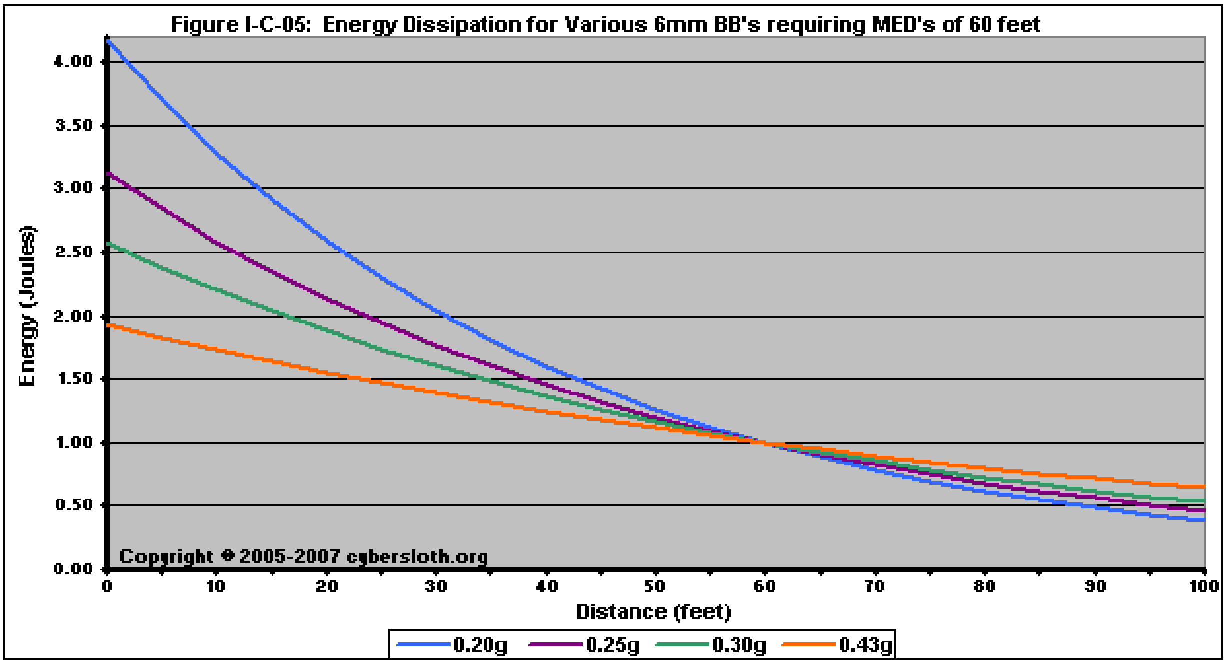
If we were to use this plot to obtain safe engagement distance (again, determined as the point at which the BB has less than 1.00 J of kinetic energy), we can see that the 0.20g BB would need an MED of about 17 feet, while the 0.25g BB needs an MED of about 21 feet. This may not seem significant, however when talking about rifles with very high muzzle energies, the discrepancy in MED between BB's of different masses becomes tremendous.

Figure I-C-04 depicts three BB's all fired at the same muzzle energy of 3.35 J, the equivalent of 600 fps with 0.20g BB's:



In this case, the 0.20g BB will have dissipated enough energy such that it is safe at a distance of about 51 feet, while the 0.30g and 0.43g BB's have reached a safe energy at a distance of about 76 feet and about 109 feet, respectively. The significance of this is discussed in great detail in Section VI-C: Recommended Universal MED's.

Another example is to look at muzzle energies that require an MED of 60 feet, as depicted in Figure I-C-05. Again, the impact energy depends as much upon the weight of the ammunition used as it does muzzle energy.



To put those numbers into context, let's look at what the values correspond to when it comes to the chronograph.

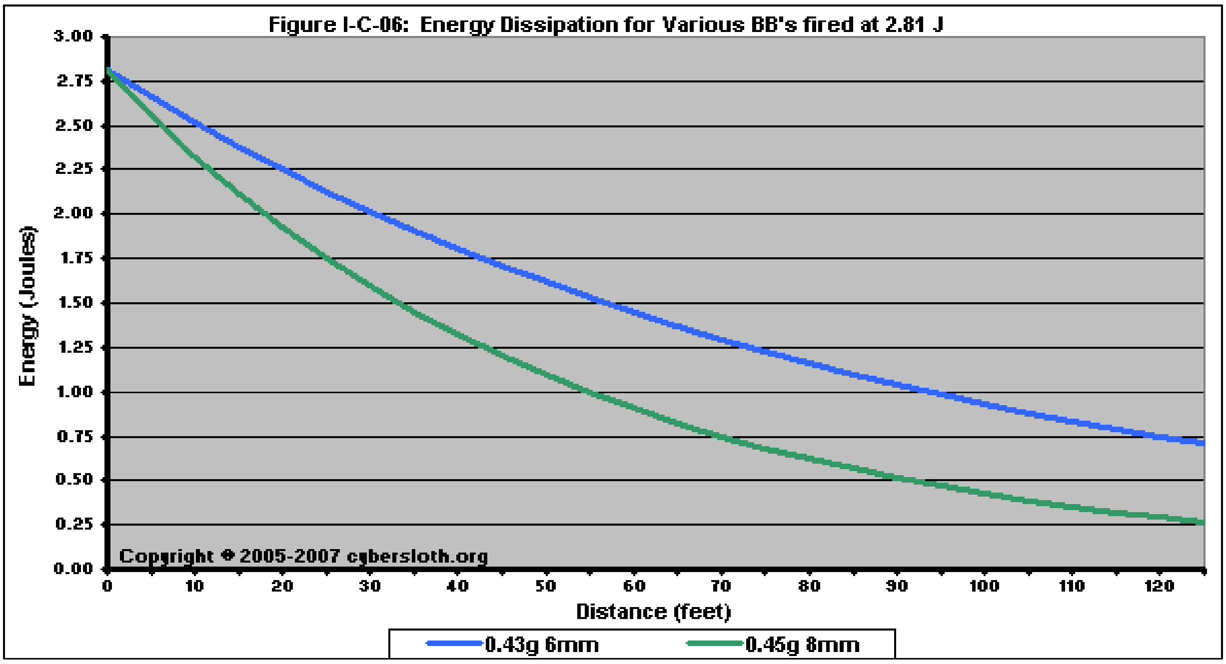
Weight of Ammo to be Used	0.20g	0.25g	0.30g	0.43g
Muzzle Energy (Joules)	4.2	3.1	2.6	1.9
Chrono Velocity w/ 0.20g BB's	670	580	527	456
Velocity for Given Ammo	670	519	430	311
MED Required	60	60	60	60

If a person arrived at a site with a rifle that chronos at 550 fps (w/ 0.20g BB's) and plans to use 0.30g BB's at the match, he or she would more than likely be turned away. If another person arrived with a rifle that chronos at 480 fps (w/ 0.20g BB's) and plans on using 0.43g BB's during the match, according to many site's rules, he or she would be allowed. In reality, the person who's rifle chrono'd at the higher velocity (550 / 0.20) would actually be safer on the field due to the weight of the ammo being used (0.30g).

The point is, and it is nonnutritive yet very important, is that **when determining velocity limits and MED's, it is just as important to restrict ammunition weight as it is to restrict muzzle velocity.** At our site, we found it more important to raise the velocity limits used by snipers and instead place a hard restriction on the weight of the ammunition used (anything *over* 0.30g is not allowed).

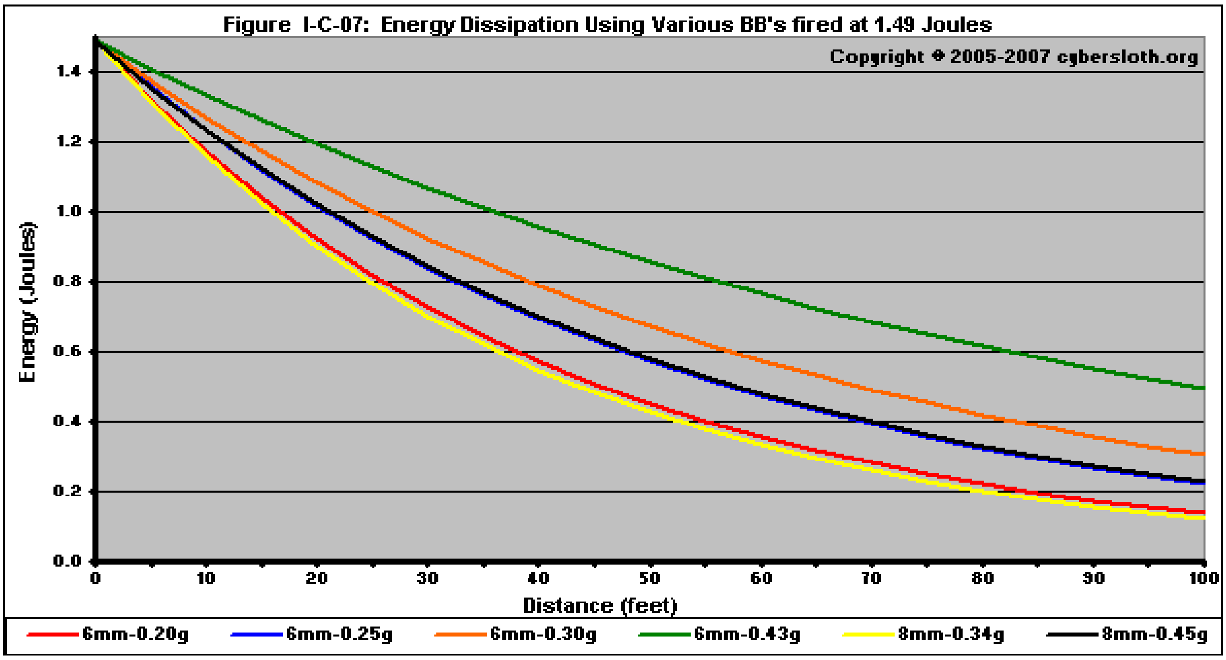
Another note of interest is that drag is affected by the size of the projectile -- specifically its orthogonal, or cross-sectional area -- such that the force of drag increases directly proportional the increase in frontal area. It makes sense, then, to realize that 8mm BB's will experience greater drag when compared to 6mm BB's at equal mass and velocity.

Figure I-C-06 shows the energy dissipation for 8mm BB's of nearly equal mass:



2.81 Joules, a muzzle energy equal to 550 fps with a 0.20g BB, corresponds to a muzzle velocity of 375 fps with the 0.43g 6mm BB and 367 fps with the 0.45g 8mm BB. Even though the 8mm BB is slightly heavier, it ultimately dissipates its energy at a considerably faster rate than the 6mm BB. In terms of safe engagement distances, the 6mm BB would need an MED of about 94 feet, whereas the 8mm BB would only need an MED of 57 feet.

Figure I-C-07 depicts the energy dissipation for various BB's, all fired at 1.49 Joules (the equivalent of 400 fps with 0.20g BB's):



Notice that the 0.43g 6mm BB retains its energy best. The 8mm BB's dissipate energy at a rate similar to that of significantly lighter 6mm BB's.

Comparing the 6mm BB's to 8mm BB's, we see that 0.34g 8mm BB's dissipate energy at about the same rate as 0.20g 6mm BB's, and 0.45g 8mm BB's dissipate energy at about the same rate as 0.25g 6mm BB's (the 0.25g 6mm BB line is hard to see as it is overlapped by the 0.45g 8mm BB line). And all four of those BB's dissipate energy faster than any of the heavier 6mm BB's (>0.26g). It might seem as though density is the governing factor for why certain BB's dissipate energy faster than others. However, there is no direct-correlation between energy dissipation and density. Rather, energy dissipation is related to velocity dissipation, which is a function of initial velocity, frontal area, and mass.

One additional point concerning 8mm BB's is that their inherently safer than 6mm BB's at equal impact energies. This goes back to the original statement that it is the impact energy in relation to impact area is ultimately what determines whether or not a projectile will penetrate a given target. By increasing impact area (and an 8mm BB has nearly twice the impact area of a 6mm BB), the impact energy is distributed over a larger area. To many, this seems counterintuitive but it explained in further detail in Section VI-B: Safe Impact Energy.

ATP - Forces Governing Trajectory

Section I-D: Forces Governing Trajectory

All of the equations listed below are in algebraic form, rather than integrals. This is done in order to present the physics in a easily graspable form.

For all of the calculations I made, I took the axis orientation such that the x-axis dealt with the forward/backward direction of the trajectory (i.e., parallel to the ground), the y-axis dealt with the up/down direction (i.e., perpendicular to the ground), and the z-axis dealt with any lateral movement of the BB (such as due to a crosswind). That is to say that I broke everything up into vector components, although the equations do not reflect this.

Section I-D-01: Drag Force

The force of drag $\mathbf{F_D}$ can be calculated using the following formula:

> $$F_D = {}^1/_2 * C_D * r * A * v^2$$

where $\mathbf{C_D}$ is the drag coefficient. For a non-rotating sphere, $\mathbf{C_D}$ is constant. I originally estimated $\mathbf{C_D}$ to be 0.47; however this is not a static number for the analysis as the projectiles are never in a true non-rotating state. In reality, $\mathbf{C_D}$ tends to fall somewhere between 0.42 and 0.50, depending on the amount of spin imparted. For determining drag coefficient, see below (Section I-D-07: Drag Coefficient),

r (rho) is the air density in $^{kg}/_{m}3$ (as discussed in Section I-B: Air Density),

A is the cross-sectional area of the BB. In this case, it's simply calculated as a circle's area with a diameter of 6mm. For my calculations, I came up with A = 0.000028274 m^2 = 0.000304314 ft^2 = 0.043825132 in^2

v is the instantaneous velocity of the projectile. Because velocity is derived from the previous drag calculation, an error is introduced. The best way to minimize this error is to use very, very small time intervals for calculating velocity, as well as using and ODE approximation such as the Runge-Kutta method. At time intervals of 0.001 seconds, I found that the error was completely nullified (i.e., by comparing results of velocities calculated using pre- and post-velocity results -- the results were less than 0.1 fps at 150').

Additionally, it is useful to have the equation for calculating Reynolds number for the projectile:

Re = D * r * v * u

where **D** is the diameter in meters,

r is the air density,

v is the instantaneous velocity, and

u (nu) is the air viscosity (which is 17.4 x 10^{-6} Pa*s at STP)

Again, I originally estimated a $\mathbf{C_D}$ of around 0.47. At low speeds, $\mathbf{C_D}$ is slightly lower, while at high speeds it increases to around 0.5. Technically speaking, a smooth sphere can surpass a critical velocity wherein the $\mathbf{C_D}$ would be greatly diminished. This is most probable for larger objects, however, and is considered impossible in the realm of airsoft. (For this to happen with a 6mm BB, it would have to moving several times faster than 0.308 bullet.)

Ultimately, $\mathbf{C_D}$ tends to stay between 0.43 and 0.47, with $\mathbf{C_D}$ being around 0.47 for BB's with little to no spin, and about 0.43 for BB's with high spin. It is interesting to note that hop-up actually reduces drag when significant spin is applied. For aerodynamic shapes, lift incurs greater drag by inducing early separation of the boundary layer. However, for the rather non-aerodynamic sphere, spin actually increases laminar flow resulting in lowered pressure drag. This is explained in Section III: Effects of Hop-Up. While calculations are continuously made throughout the model run to determine C_D accurately, I later found that such calculations were not paramount for determining trajectory nor energy dissipation. Model runs with a constant C_D of 0.43 showed a similar trajectory to model runs using a CD of 0.47. Further, the velocity difference at 100 feet when using the two disparate drag coefficients was usually less than 5 fps (and the energy difference was essentially negligible).

Section I-D-02: Velocity

To determine velocity, the following equation is used:

$$\mathbf{v_f = v_i + a * t}$$

Here, the resultant velocity $\mathbf{v_f}$ is calculated by taking the previous velocity measurement $\mathbf{v_i}$ and adding it to the velocity change due to deceleration. Pretty straightforward in terms of physics equations. Do note that the acceleration is the *average* acceleration so, again, using small increments are necessary to minimize any errors.

One thing that should be noted is that velocity changes in a non-linear fashion. What this means is that if you measure your muzzle velocity as 300 fps and find that at 40 feet the BB is moving at 200 fps, you cannot assume that the BB is moving at 250 fps at 20 feet. This is because the velocity curve represents an exponential decay. Reality for the given example is that at 20 feet, the BB would be moving at around 235 fps.

Think of it this way: for a gun firing at 300 fps with a 0.20g BB, at 5’ the BB has slowed by 18 fps. While traversing the distance from 5’ to 10’, it only slows 16 fps, and during the 10’ to 15’ interval, it only slows 15 fps. Once the BB approaches a speed under 100 fps, it’s only losing about 4 fps for every 5’ traveled. Granted, it’s moving so slow at that point that it’s actually moving downward faster than it is moving horizontally.

Velocity at range is different for BB's shot with and without hop-up. BB's with high spin will experience less drag and will ultimately have a higher velocity downrange. This velocity difference is small, however it is noticeable. At 100 feet, a spinning BB may be moving as much as 15 fps faster than a BB with little or no spin. All velocity listings in Section VIII are for BB's with no spin.

Section I-D-03: Distance

To determine distance, the following equation is used:

$$\mathbf{x_f = x_i + v_a * t + {}^1/_2 * a * t^2}$$

Here, distance is incrementally calculated where resultant distance $\mathbf{x_f}$ is calculated by taking the previous distance measurement $\mathbf{x_i}$ and adding it to the average velocity and acceleration change. Again, pretty straightforward in terms of physics equations. Do note that here both velocity and acceleration are the *averages* so, again, using small increments are necessary to minimize any errors, as well as ordinary differential equation techniques.

Section I-D-04: Magnus Force

To calculate Magnus force, I used the following equation:

$$\mathbf{F_M = C_L * r * v^2 * A}$$

where $\mathbf{C_L}$ is the lift coefficient (explained in <u>Section I-D-08: Lift Coefficient</u>),

where **r** is the air density in $^{kg}/_{m}3$,

where **v** is the *average* velocity in meters per second,

and where **A** is the cross-sectional area of the projectile.

Keep in mind that the force is going to be orthogonal to the velocity.

Section I-D-05: Terminal Velocity

Terminal velocity is the maximum velocity an object can reach in freefall through atmosphere. It is calculated by determining what velocity (in the y-, or up/down, axis) is necessary to create enough drag such that drag force (again, in the y-direction) is equal to the force of gravity.

$$\mathbf{v_t = ((F_g) / ({}^1/_2 * C_D * r * A)) \hat{}\ {}^1/2}$$

For example, a 0.20g 6mm BB would have a terminal velocity of about 35 mph (or 52 fps) at sea level at room temperature.

Section I-D-06: Spin Decay

Spin decay is the rate at which a solid, spherical object, slows down from a given rotational velocity. For instance, a CD-ROM might have a CD spinning at 15,000 rpm (revolutions per minute). If the CD-ROM were turned off (and the brake disabled), it might take a minute for it to stop spinning with only air friction working to slow it down (there would be mechanical friction, but this is a hypothetical situation). Spin decay governs how quickly factors such as air friction act to overcome rotational inertia and cause the spin imparted upon a BB by the hop-up mechanism to degrade.

To determine spin decay, it's necessary to determine how much toque is induced by air friction. Unfortunately, this is one of the difficult things to determine and some estimations had to be made by looking at trajectories of BB's from a side view. While calculating torque is simple, calculating the friction coefficient is not. BB's, like bullets, exit the barrel with a high amount of spin (though BB's are not nearly as high as bullets). Unlike bullets, which have significant rotational inertia in comparison to the torque induced by air friction, the spin rate of the BB begins to degrade rather rapidly. This is one section that I'll have to come back to once I've determined the actual constants to use for spin decay.

In the interim, I've used modified torque equations to estimate the effects of hop-up. While the estimations are close to reality, they're not as accurate as I'd like (simply because they've been derived from empirical observations as opposed to straight physics). I'm going to hold off on posting the complete set of equations until I've had a chance to fill in all of the coefficients. Once I've determined the proper equations and coefficients, I'll post the estimated constants as well as the correct ones, and will replot the charts *only if necessary*. I will add that if you're looking to perform graduate work in mechanical or aeronautical engineering and haven't found a topic, there is a dearth of information concerning spin decay for spherical objects. Hint, hint.

For the time being, the current calculation for spin decay depicts trajectories close to reality. Further, the results are very close to other calculations for relatively smooth spheres experiencing high Reynolds numbers (though there is a fourth source that seems to disagree with the other three).

In terms of simplified equations for determining spin decay, the following equations were used:

The angular acceleration **a** is calculated as

$$\mathbf{a = t / I}$$

where **t** is the torque and **I** is the moment of inertia for a solid sphere.

Torque is calculated as:

$$\mathbf{t = {}^{1}/_{2} * C_T * r * r^{3} * w^{2}}$$

where $\mathbf{C_T}$ is the torque coefficient,

r is the radius of the sphere,

and **w** is the angular velocity.

Thetorque coefficient $\mathbf{C_T}$ is generally calculated as

$$\mathbf{C_T = 6.45 / ((Re_w)\text{^}{}^{1}/2) + 32.1 / Re_w}$$

where $\mathbf{Re_w}$ is the Reynolds number for centerline rotation.

The Reynolds number for centerline rotation $\mathbf{Re_w}$ is generally calculated as

$$\mathbf{Re_w = r * r^{2} * w / h_f}$$

where $\mathbf{h_f}$ is the viscous friction coefficient.

Section I-D-07: Drag Coefficient

As stated earlier, the drag coefficient is not a static number. As the rotational velocity **V** changes with respect to linear velocity **U**, so does the drag coefficient. Both the drag coefficient and lift coefficient have been studied and determined, most notably through the research of Achenbach* and Mehta**. Fortunately for us, Dr. Gary Dyrkacz has has taken the older plots and, using SigmaPlot, determined the polynomials necessary to calculated both $\mathbf{C_D}$ and $\mathbf{C_L}$ using data from Davies' study of golf balls***.

(If you get a chance, visit Dr. Dyrkacz's page on The Physics of Paintball as his page describes in detail what happens to a projectile such as a spinning paintball as it moves through the air, as well as providing the calculus-based equations which are more useful to us even though they're much harder to type!)

$\mathbf{C_D}$ is initially calculated without spin as $\mathbf{C_{D0}}$, and is determined by the equation:

$$C_{D0} = (0.4274794 + 0.000001146254 * Re - 7.559635 \times 10^{-12} * Re^2 - 3.817309 \times 10^{-18} * Re^3 + 2.389417 \times 10^{-23} * Re^4) / (1 - 0.000002120623 * Re + 2.952772 \times 10^{-11} * Re^2 - 1.914687 \times 10^{-16} * Re^3 + 3.125996 \times 10^{-22} * Re^4)$$

Where **Re** is the Reynolds Number.

With spin, $\mathbf{C_D}$ is calculated using the equation:

$$C_D = (C_{D0} + 2.2132291 * {}^V/_U - 10.345178 * ({}^V/_U)^2 + 16.157030 * ({}^V/_U)^3 - 5.27306480 * ({}^V/_U)^4) / (1 + 3.1077276 * ({}^V/_U) - 13.6598678 * ({}^V/_U)^2 + 24.00539887 * ({}^V/_U)^3 - 8.340493152 * ({}^V/_U)^4 + 0.07910093 * ({}^V/_U)^5);$$

where **V** is the rotational velocity and **U** is the linear velocity.

Notice that a sphere with a high amount of spin would have a CD of around 0.43, marginally less than the 0.47 I used for non-spinning spheres.

Section I-D-08: Lift Coefficient

Again, I had to use Dr. Dyrkacz's polynomial to calculate $\mathbf{C_L}$.

$$C_L = (-0.0020907 - 0.208056226 * ({}^V/_U) + 0.768791456 * ({}^V/_U)^2 - 0.84865215 * ({}^V/_U)^3 + 0.75365982 * ({}^V/_U)^4) / (1 - 4.82629033 * ({}^V/_U) + 9.95459464 * ({}^V/_U)^2 - 7.85649742 * ({}^V/_U)^3 + 3.273765328 * ({}^V/_U)^4);$$

Section I-D-09: Gravity

Gravity can normally be assumed as a constant mass times acceleration where the gravitational acceleration is about 9.8 $^m/_s$ 2. However, gravity can be more accurately calculated (and was done so in the model) by using the following equation:

$$A_g = 9.7803185 * [1 + (0.005278895 * (\text{sine (Lat)})^2) - 0.0000589 * (\text{sine} (2 * \text{Lat}))^2)]$$

where **Lat** is the latitude in degrees.

The acceleration due to gravity varies from about 9.78 $^m/_s$ 2 at the equator to about 9.83 $^m/_s$ 2 at the poles. In truth, while the model did take into account the variation of the gravitational acceleration in accordance with latitude, it was not necessary to do so. The effects of gravity at various latitudes has a miniscule effect on trajectory; using a constant of 9.8 $^m/_s$ 2 would have been sufficient.

* "Experiments of the flow past spheres at very high Reynolds numbers," Achenbach. E., in American Journal of Physics, 54, 565-575 (1972).

** "Aerodynamics of sports balls," Rabindra D. Mehta, in Annual Review of Fluid Mechanics, 17, pages 151-189 (1985).

*** Davies, J.M., The Aerodynamics of Golf Balls, Journal of Applied Physics, 20, pages 821-828.

The Airsoft Trajectory Project

ATP - Testing and Model Validation

Section II: Testing and Model Validation

Before publishing the calculations, I wanted to make sure that the equations I was using came fairly close to real world results. This section concerns the testing methods I used to verify the calculations.

Section II-A: Verifying Velocity Calculations

After I'd created the program I set about testing to make sure reality matched up with my theoretical calculations. I fired airsoft BB's from guns with muzzle velocities ranging from 200 fps to 600 fps. I performed thorough tests measuring velocity at 5' intervals for several different guns with the maximum range in most cases being 60' (though in the case of the 500+ fps velocities, I tested at 10' intervals out to a maximum range of 100').

Also, I asked for data from other sources, people who'd collected data at different temperatures and different altitudes. I also tested the data using different size BB's (in this case, 8mm BB's and paintball BB's). For all of the data collected, I compared it to the calculations made by the program. For data collection, at each range I needed a minimum of 20 data points. Basically, the more data collected would yield a better average velocity.

When comparing theoretical velocity calculations to actual results, my goal was for the program's calculations to be within two standard deviations of the data collected. For those who haven't done much statistical work, a "standard deviation" is a statistical method for calculating the probability that sample data will lie within a normal distribution. For example, if I collected fifty data points for velocity and the average velocity was 243.1 fps with a standard deviation of 2.0 fps, then 68% of the data points will be within one standard deviation of the average (i.e., 68% of the measurements will be between 241.1 and 245.1 fps). Furthermore, two standard deviations encompass 95% of the data, while three standard deviations encompass 99.7% of the data. In other words, if I fired over 1,000 shots and the average was 243.1 fps with a standard deviation of 2.0 fps, 997 of the shots would have been between 237.1 and 249.1 fps.

Having said that, I found that the results I had calculated matched to within two standard deviations of the collected data, with most measurements being well within one standard deviation. Considering that the standard deviation was usually less than 2.0 fps, I would say that the calculations are very, very accurate.

Here's an example of how I compared the theoretical calculations to collected data. I used a Chrony F-1 and a Chrony Beta Master as the test platforms, with the F-1 sitting 1-2 inches in front of the muzzle, and the Beta Master down range. Additionally, I had the gun sitting in a vice (which took some tweaking in order to line it up with the chronograph). On each of the days I took the measurements, I recorded the temperature and used it for making the calculations, with some of the measurements being taken indoors and some being taken outdoors in excellent weather (though I waited for windless days to do the outdoor work). Additionally, most of the outdoor work was done under a pavilion which further minimized any weather effects.

.	PSG-1	0.30g	.		M-4	0.25g	.
.	0'	20'	50'	70'	0'	15'	30'
Calculated	516.3	439.2	345.1	293.3	290.6	251.5	217.4
Observed	516.3	440.0	342.9	294.6	290.6	254.1	215.1
Std. Dev	-	4.7	4.1	5.2	-	1.7	2.8

Notice that the calculated data is well within two standard deviations of the measured results. In most cases, the test data was within one standard deviation of the calculations. The test data deviated by more than one standard deviation for only two out of more than thirty datasets.

Keep in mind that these measurements were taken both in and outdoors. While a crosswind will affect the BB's trajectory laterally, *it will not alter the forward velocity of a BB in flight*. Even so, testing was only done on completely calm days.

One thing that has been asked before is whether misalignment of the chronograph or rifle could lead to errors, as well as lighting conditions. When using the chronograph, I was always in the shade and used the lighting attachments for consistency. The same lighting attachments were used indoors. Each time the chronographs were set up, they were also tested to make sure that they were registering readings that were consistent with one another (clocked using an AEP that consistently fired shots within 1% of average).

As for alignment, one of the guys I was working with asked if we needed to slant the chronographs to take into account the fact that the BB was moving both forward and downward when it passed through the down range gates. Actually, it is better to keep the chronograph level as we were only concerned with measuring velocity along the horizontal axis -- the vertical velocity would be completely dependent upon gravity and is easily calculated.

Additionally, the gun could be aimed off-angle to the chronographs. While it is easy to notice a misalignment that is five or more degrees, it's harder to see a misalignment of only one or two degrees. Fortunately, this type of misalignment does not contribute to great inaccuracies. Chrony chronographs are used by the Sheriff's Department here, and we were discussing with them bullets that were fired at around 2800 fps. A five degree misalignment would only introduce a 10 fps error, meaning that we would measure 2790 fps instead of 2800. If a rifle that normally fires 0.20g BB's at 325 fps were misaligned by 5 degrees, the erroneous reading would be only 324 fps -- considering that most rifles shoot over a range at least as wide as +/- 3 fps, a 1 fps difference is acceptable. Even a misalignment of 10 degrees, which is ridiculously noticeable, would show the 325 fps rifle as shooting at 320 fps -- quite an error but, again, if you aren't able to visually notice a 10 degree misalignment, you should probably put down the rifle and visit an ophthalmologist.

(Just as a side note, I do want to stress that at the latter distances, it was *very* difficult to take measurements. I found that at extreme distances -- greater than 50' or in the case of the PSG-1, greater than 80' -- I would only get a reading on the chronograph about once every 1-2 shots. If anyone makes similar measurements, please let me know. If you do collect some data, please also let me know what type of chronograph you were using, what altitude you were at, and the ambient air temperature.)

The Airsoft Trajectory Project

ATP - Effects of Hop-Up

Section III: Effects of Hop-Up

Friction interaction between the hop-up bucking and pellet incur a backspin upon the pellet. This backspin induces force orthogonal to the pellet's motion due a special condition of Bernoulli's Principle known as the Magnus Effect. The Magnus Effect is due to the high-speed interaction between the pellet and the surrounding air. (For simplicity, spin hereafter refers particularly to backspin.)

There is often debate over which force is more so responsible for the effect of lift, Bernoulli's Principle or the Magnus Effect. Technically, Bernoulli's Principle deals with low pressure due to an object moving over a medium at high speed (or the medium moving over the object). Magnus Lift is more accurately attributed to the momentum shift of the boundary-layer due to variable areas of low pressure around the object, the effect of which causes a net movement of the object in the direction opposite of the motion of the boundary layer. Both cause lift on the BB's, however the effects of each are significantly dependent upon 1) the speed of the object through a medium, and 2) the rotational velocity of the object.

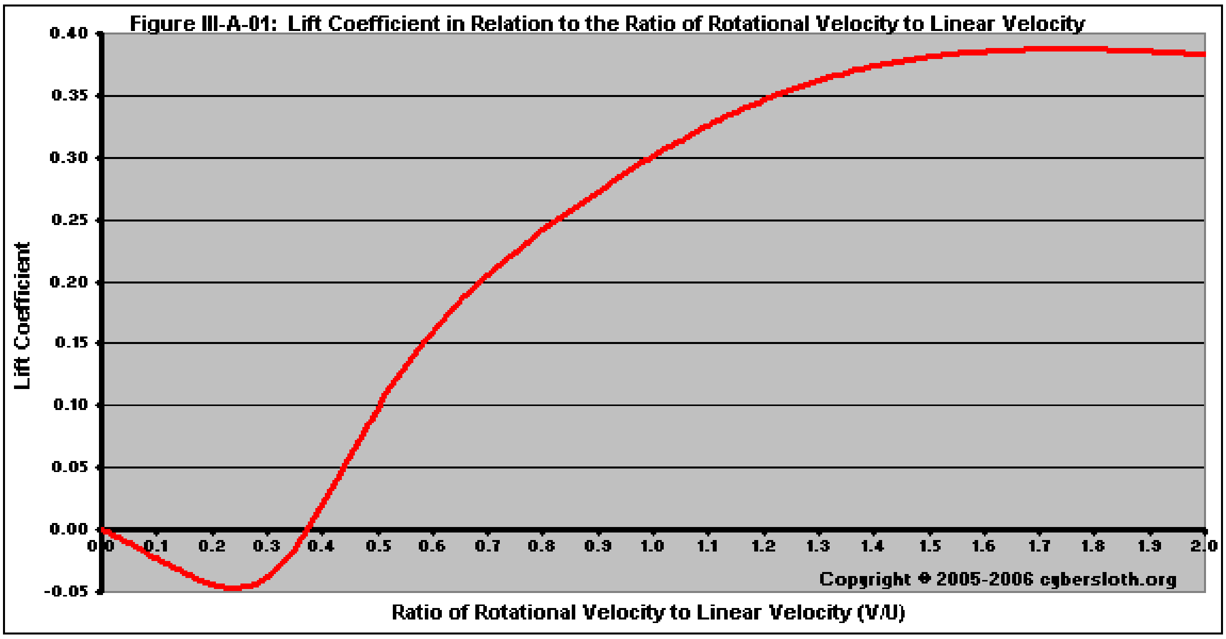
When a relatively smooth, spherical object spins at a low rate in relation to its forward motion, the surrounding air is fairly laminar. Stable laminar flow induces early separation of the boundary layer leading to a large wake field which induces an area of low pressure immediately behind the ball (incurring what is known as pressure drag). Not only does this affect the overall drag experienced by the sphere, smooth laminar flow has an adverse affect on lift as well. While backspin causes an area of low pressure above the sphere due to Bernoulli's Principle, a relatively low rotational velocity leads to an upward shift in the boundary layer, negating the lift due to Bernoulli's Principle and ultimately causing negative lift. This is known as the Reverse Magnus Effect. Consequently, low spin rates provide two drawbacks to trajectory: higher drag and reverse lift.

When the spin rate is high in relation to the forward motion, the surrounding air becomes turbulent. Turbulent airflow -- which normally increases drag on aerodynamic objects -- actually serves to increase airflow over the relatively non-aerodynamic sphere causing the boundary layer to follow the contour of the sphere's surface longer. (This is the same reason that golf balls have dimples, to induce turbulent flow which ultimately lowers the Reynolds number.) The turbulent airflow results in minimizing the wake field and ultimately diminishing the pressure drag. Additionally, the spin causes the boundary layer to experience a downward motion relative to the projectiles position. This leads to a net force of lift upon the sphere as the momentum shift of the boundary layer known as Magnus Lift. The higher the spin rate in relation to forward motion, the greater the area of lower pressure above the sphere and the greater the momentum shift, and thus the greater the effect of Magnus Lift.

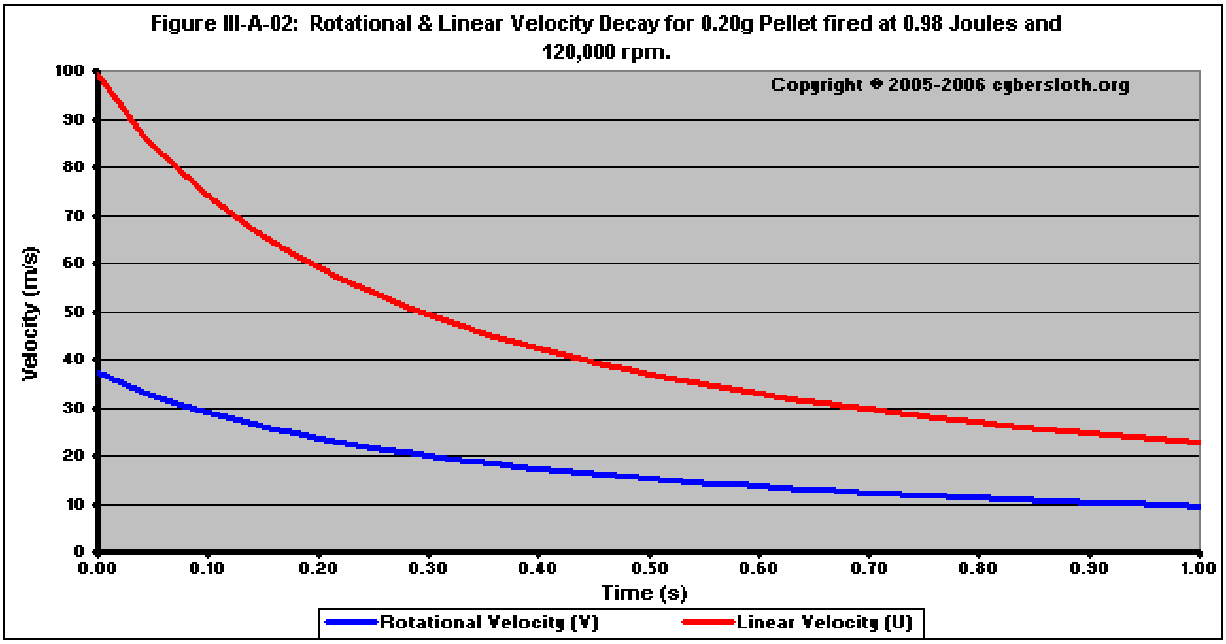
Upon leaving the barrel, the pellet has a specific velocity and a specific rate of spin, which are translated into linear velocity (U) and rotational velocity (V), respectively. Additionally, both velocities begin exhibiting exponential decay as the linear velocity decreases due to the drag while the rotational velocity decreases due to surface friction. As the linear velocity decreases at a greater rate than that of the rotational velocity, the ratio of rotational velocity to linear velocity, V/U, grows.

A perfect friction exchange between the pellet and hop-up bucking would result in an initial V/U of 1. Although the friction exchange is very efficient, it simply is not possible to have a perfect friction exchange. Furthermore, the pellet has not reached its peak velocity as it makes contact with the hop-up bucking early in its transit through the barrel (though the rotational velocity will increase slightly as it spins down the barrel). For the majority of calculations, V/U was capped at 0.41. For a 0.20g pellet fired at 328 fps (or the equivalent of 1.00 Joule), a V/U of 0.41 would translate to a spin rate of 132,000 revolutions per minute. By comparison, a typical bullet leaves the barrel spinning at anywhere from 200,000 rpm to 300,000 rpm. For bullets, spin is directly proportional to the linear velocity and is easily calculated by multiply the muzzle velocity by the twist ratio. For instance, a .308" bullet might have a muzzle velocity of 2800 fps out of a rifle with a twist ratio of 1:8. In this case, it could have a spin of over 250,000 rpm. (Coincidentally, many bullets begin to suffer from instability issues due to structural failure of the jacket above 400,000 rpm.) While bullets are moving at much higher velocities than pellets, the pellets are significantly less dense and have a lower moment of inertia than that of bullets, which is why pellets are able to have such high spins imparted at relatively low velocities.

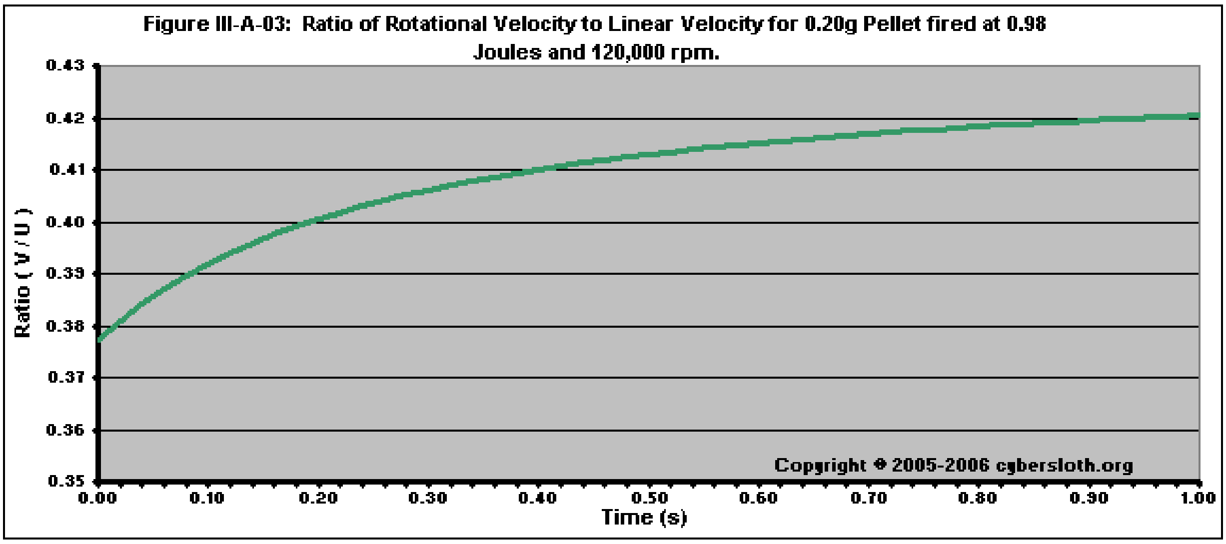
Figure III-A-01 illustrates the amount of lift generated proportional to the ratio V/U. Below a V/U of 0.37, the lift coefficient is actually negative -- anything below a ratio of 0.37 is subjected to the Reverse Magnus Effect. Above 0.37, lift is positive. Even if the pellet were to leave the barrel at a V/U less than 0.37, the linear velocity generally decreases at a greater rate than the rotational velocity, causing V/U to increase as the pellet follows its trajectory. If the rotational velocity were constant, the pellet would arc high into the air. An ideal hop-up setting will try to achieve lift coefficient that grows in relation to gravity (yet is not so high as to cause a high arc) thereby providing a seemingly flat trajectory.



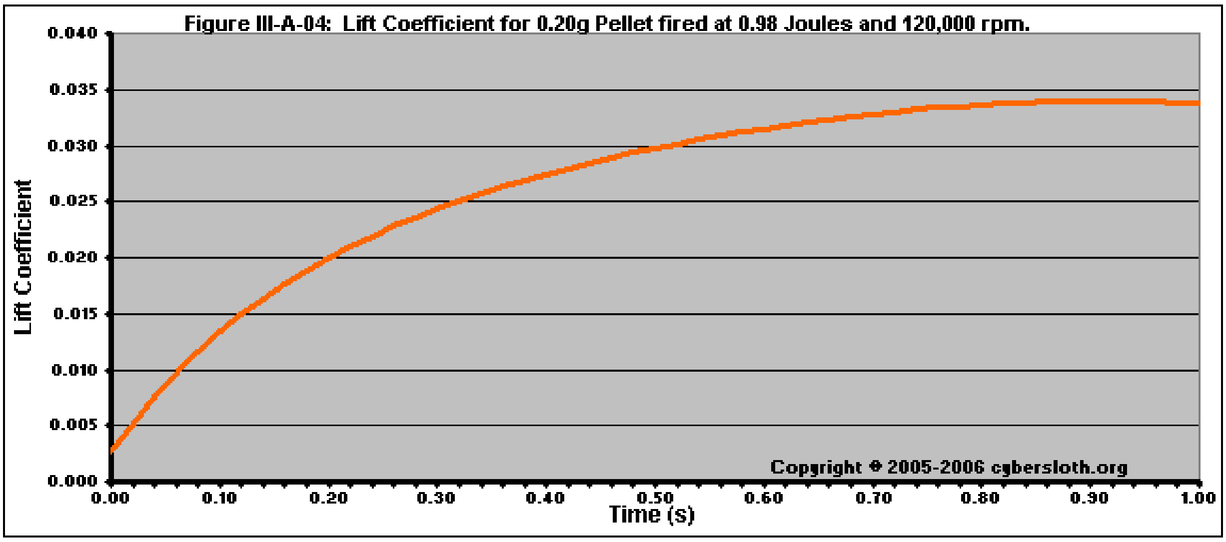
To explain how V/U affects the net vertical movement of the pellet, I have generated four plots that show how rotational velocity (V), linear velocity (U), the lift coefficient (C_L), and net force experienced by a 0.20g pellet fired at 325fps with 120,000 rpm spin. In figure III-A-02, we can see how the linear velocity decreases rapidly (compared to the rotational velocity).



Since U decreases faster than V, we see the ratio V/U increase as depicted in Figure III-A-03.



In Figure III-A-04, we can see how V/U contributes to lift, with positive lift being generated for quite a while. While the lift coefficient does steadily increase, the forward velocity (U) is decreasing rapidly which translates to a lower lift force at distance.



In Figure III-A-05, we can see how the opposing forces of lift (F_L)and gravity (F_g)affect the overall vertical force (F_y) upon the pellet. For the first 0.04 seconds, the pellet is actually moving downward. For the next 0.26 seconds, the pellet is accelerating upwards. After 0.30 seconds (and a total distance of about 67 feet), the pellet begins a downward acceleration. Keep in mind that though the pellet is accelerating downwards, that does not mean that its velocity is downward but rather that it has a positive velocity that is quickly diminishing. The pellet does not start moving downward until after about 0.51 seconds have passed (or about 96 feet of distance).

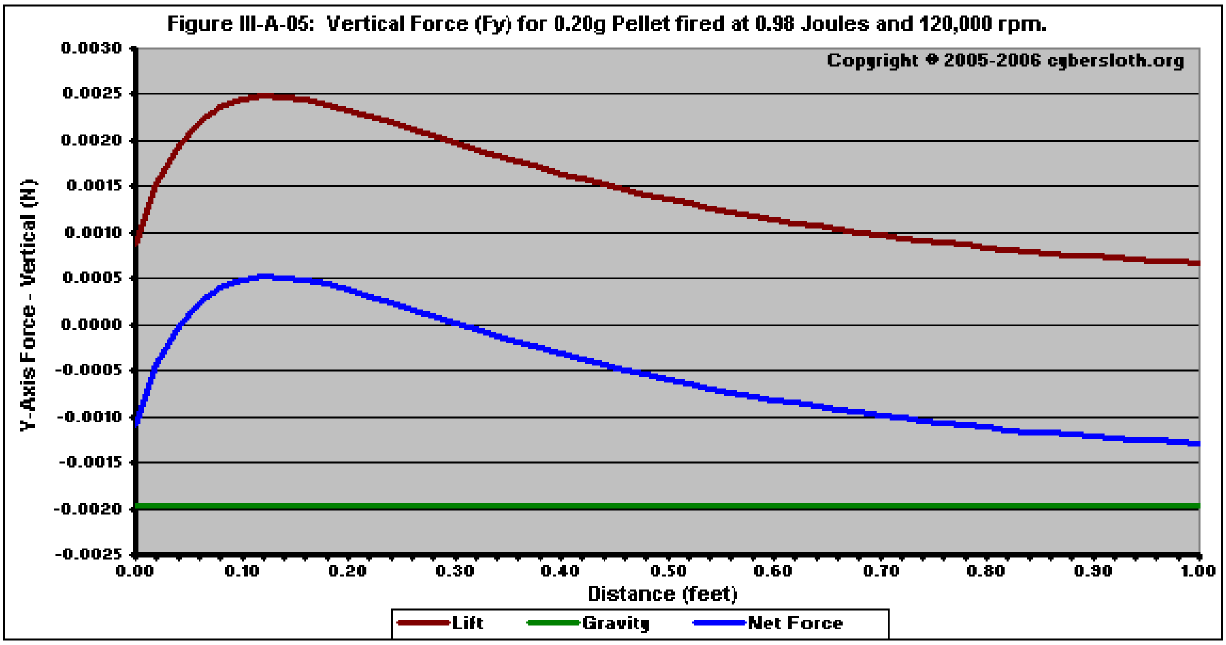
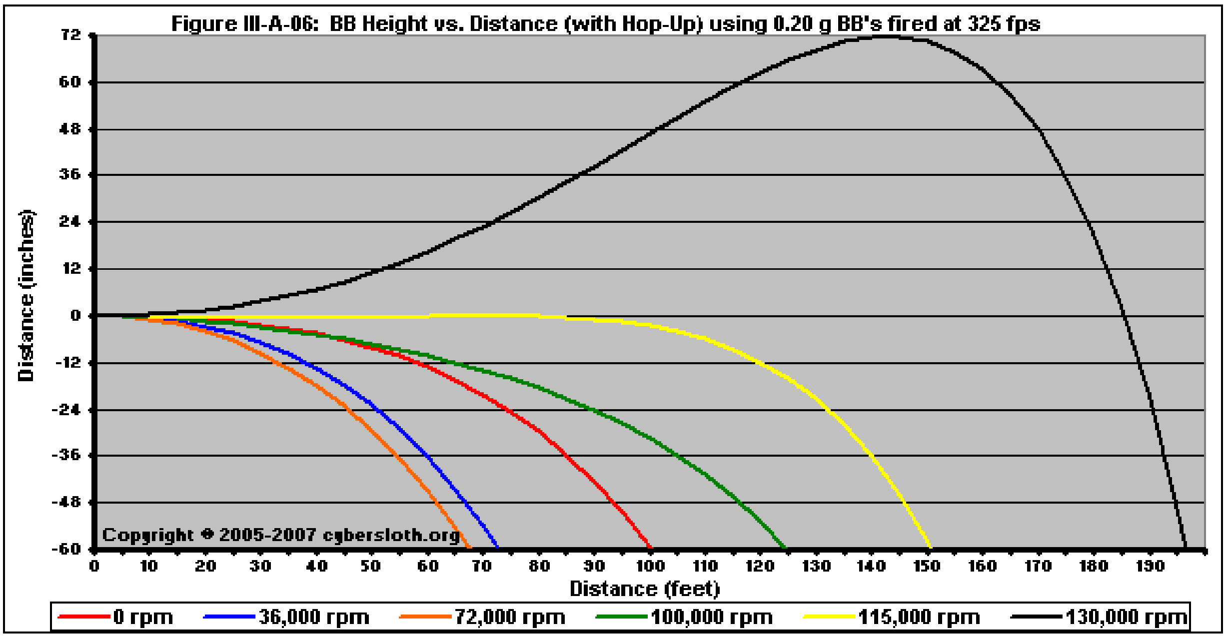


Figure III-A-06 shows the effects of hop-up upon a pellet. Notice that at 0 revolutions per minute (rpm) the pellet experiences a normal parabolic trajectory. At 36,000 rpm, V/U is low enough such that lift is negative due the Reverse Magnus Effect; since the force is negative (i.e., downward) it aids gravity in causing the pellet to fall. At 72,000 rpm, the Reverse Magnus Effect is higher still, causing a sharply downward trajectory. At around 100,000 rpm, the Magnus Effect begins to take effect causing positive lift, providing a trajectory closer to that of a BB fired without spin. At 115,000 rpm, the trajectory appears flat due to the effects of gravity being marginally negated due to the Magnus Force, while a higher rpm will cause the BB to arc skyward.



While the trajectory of the pellet appears to be nearly perfect out to 90 feet if fired at ~115,000 rpm, in reality the hop-up interaction doesn't always incur spin that is perfectly perpendicular to the ground along the z-axis (direction horizontally perpendicular to the rifle). Two reasons for this could be either the shooter holding the rifle with tilt or the pellet making a slightly off-center strike against the hop-up bucking.

Figure III-A-07 shows the effects of hop-up upon a 0.30g pellet fired at 407 fps. Again, we see the Reverse Magnus Effect cause degraded performance if hop-up is engaged yet not sufficiently high to generate a high V/U. It should also be noted that the plots are exaggerated along the y-axis -- while the degraded trajectory of shots fired with low hop-up are obvious on the plot, it would not be nearly so conspicuous in real observation. At 135,000 rpm, we see a relatively flat trajectory, while at 150,000 rpm we see a trajectory showing a greater arc above the aim point. While the 150,000 rpm shot will provide greater range, the 135,000 rpm would be more desirable as it would provide a flatter trajectory. Again, the axis is exaggerated -- the arc would not seem nearly so obvious in reality.

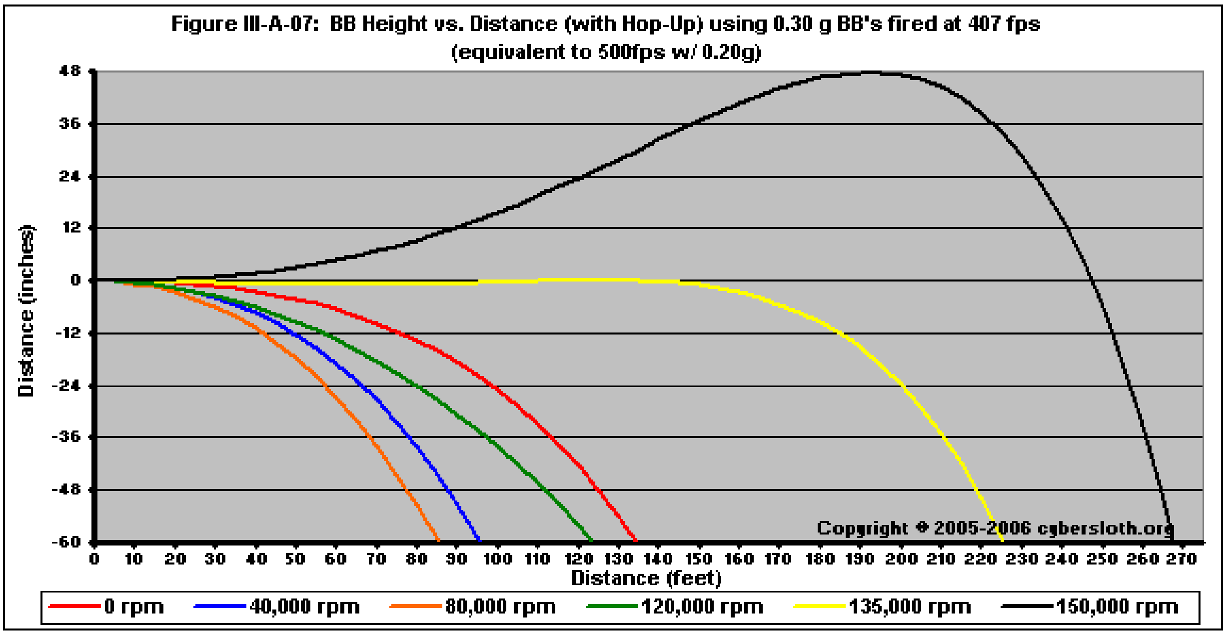
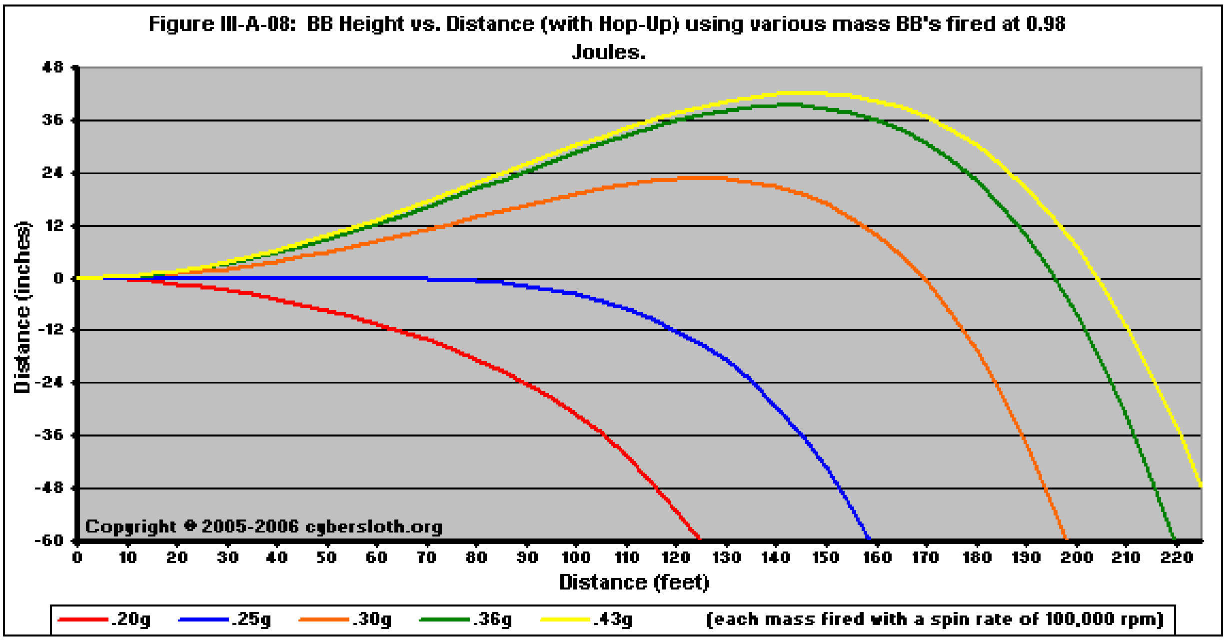


Figure III-A-08 depicts how an equal amount of spin affects various mass pellets when fired at equal energies. The reason that heavier mass pellets generate more lift is that, for equal muzzle energies, the heavier pellets have a lower linear velocity. This means that the ratio V/U is higher.



This information may seem contradictory at first. It appears as though a shooter would need to *decrease* the amount of hop-up when using heavier pellets, as opposed to reality wherein hop-up is normally *increased* when using heavier pellets. The problem lies in thinking that an unmodified hop-up setting would produce equal spin rates, however this is an incorrect assumption. Because heavier pellets will have a higher moment of inertia, more torque is necessary to generate an equal amount of spin. So while a particular setting may be fine for a 0.25g pellet, inserting a 0.30g pellet without modifying the hop-up will not generate enough torque to induce adequate spin on the 0.30g pellet. For example, a hop-up setting that results in 100,000 rpm of spin on a 0.25g pellet might generate only 80,000 rpm with a 0.30g pellet, as the heavier pellet striking the hop-up will not produce adequate torque to achieve the desired amount of spin.

Figure III-A-09 provides an estimate how a hop-up unit properly set for 0.25g pellets might produce sub par spin for the heavier pellet.

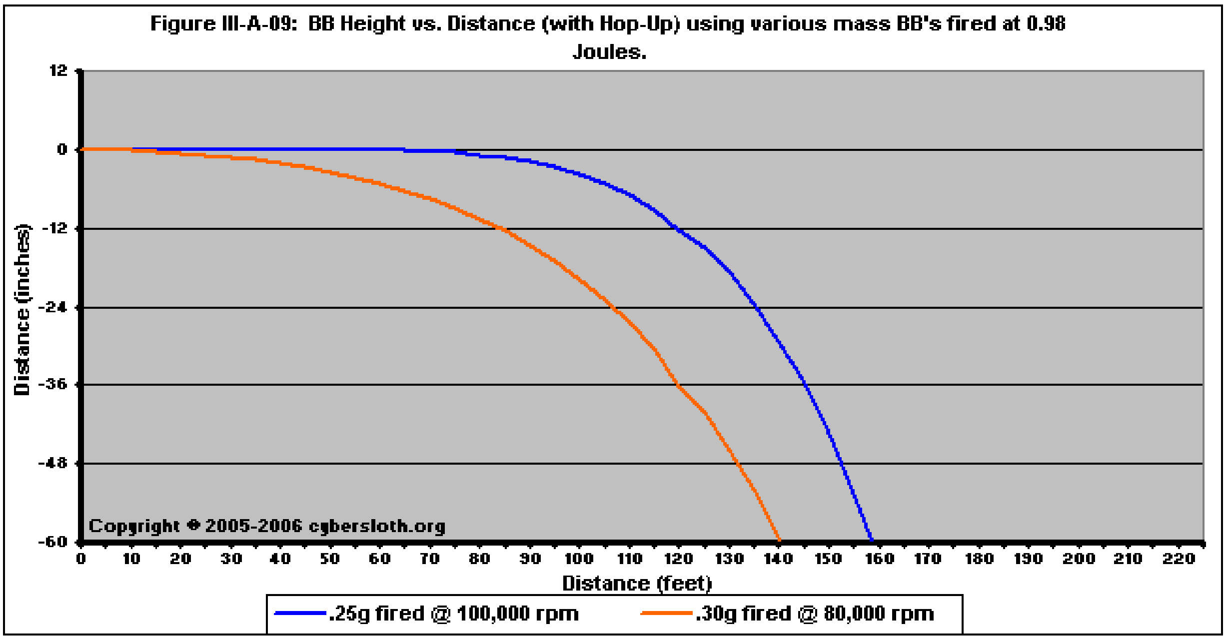


Figure III-A-09: BB Height vs. Distance (with Hop-Up) using various mass BB's fired at 0.98 Joules.

Figure III-A-10 shows how velocity affects trajectory when hop-up is applied. At lower linear speeds, V/U is high and results in a drastic, skyward arc in the pellet's trajectory. At higher linear speeds, the force of lift is insufficient as V/U is too low. It should be noted that, for equal hop-up settings, a higher muzzle velocity will result in higher spin rates (i.e., a hop-up setting that generates 95,000 rpm when a 0.20g pellet passes through at 300 fps might generate 125,000 rpm if the same pellet passed through at 400 fps), meaning that upgrading your gun will alter the trajectory, however it may not be as drastic as depicted below due to my use of consistent spin rates. I should note that a 200 fps shot with an initial spin of 95,000 rpm would have a very high V/U, and the trajectory depicted below for the 200 fps shot is probably impossible, as it would be very difficult to generate such high spin from a relatively low-energy shot.

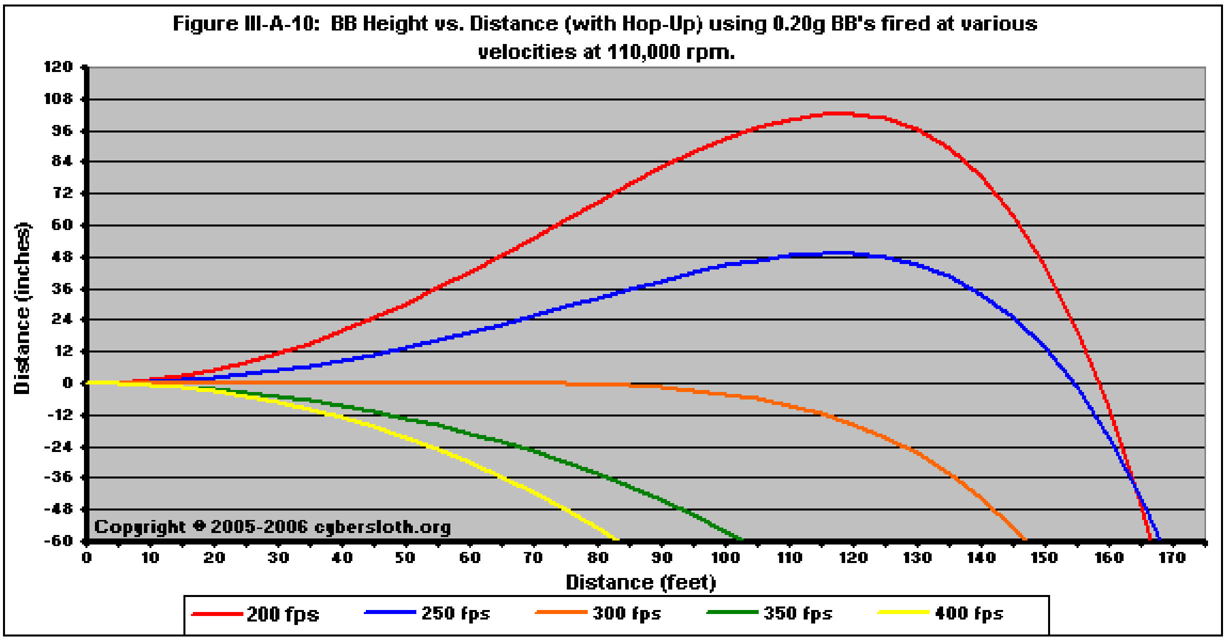


Figure III-A-10: BB Height vs. Distance (with Hop-Up) using 0.20g BB's fired at various velocities at 110,000 rpm.

Modeling trajectory in relation to hop-up proved to be a much more challenging task than any other factor contributing to trajectory. Calculating lift coefficients as well as the ratio of rotational velocities to linear velocities, and subsequently the Magnus Force, were key components that are reasonably easy to calculate. I had no luck in finding coefficients necessary to calculate the rate at which the spin drops off due to surface friction. Furthermore, collecting empirical data did not help as it was very difficult to achieve perfectly consistent results from a rifle's hop-up mechanism (most likely due to variance in muzzle energy from shot to shot), and visual observations were highly subjective; an observer would have a hard time discerning between a relatively flat trajectory and one that rose over 12 inches above horizontal at its apex.

Hop-up as modeled throughout these pages should not be taken as gospel -- they're very close but not perfect. One thing that was not taken into account in modeling hop-up is the diminished muzzle velocity due the energy loss from the pellet striking the hop-up rubber. This was initially going to be part of the model, however I could not determine direct correlation between spin and energy loss, particularly so considering that different rubbers seemed to have different effects with no uniform consistency. At any rate, while energy dissipation, velocity loss, and other things have been tested and verified, the hop-up model is theoretical and should be used as a *guide* to understand the relationship between velocity, mass, and spin; trying to use the hop-up model plots to estimate how much you should aim above a target at 200 feet is beyond the current limits of the model and should only be used with a grain of salt.

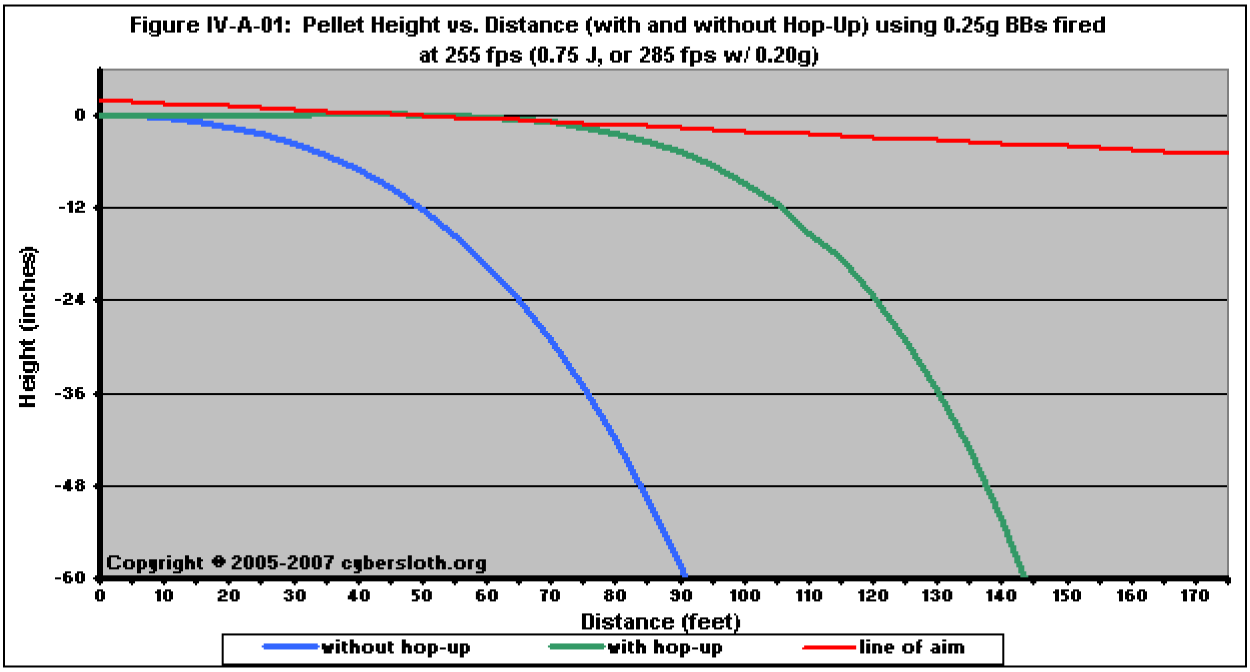
The Airsoft Trajectory Project

Effective and Maximum Range for 6mm BB's

Section IV: Effective and Maximum Range for 6mm BB's

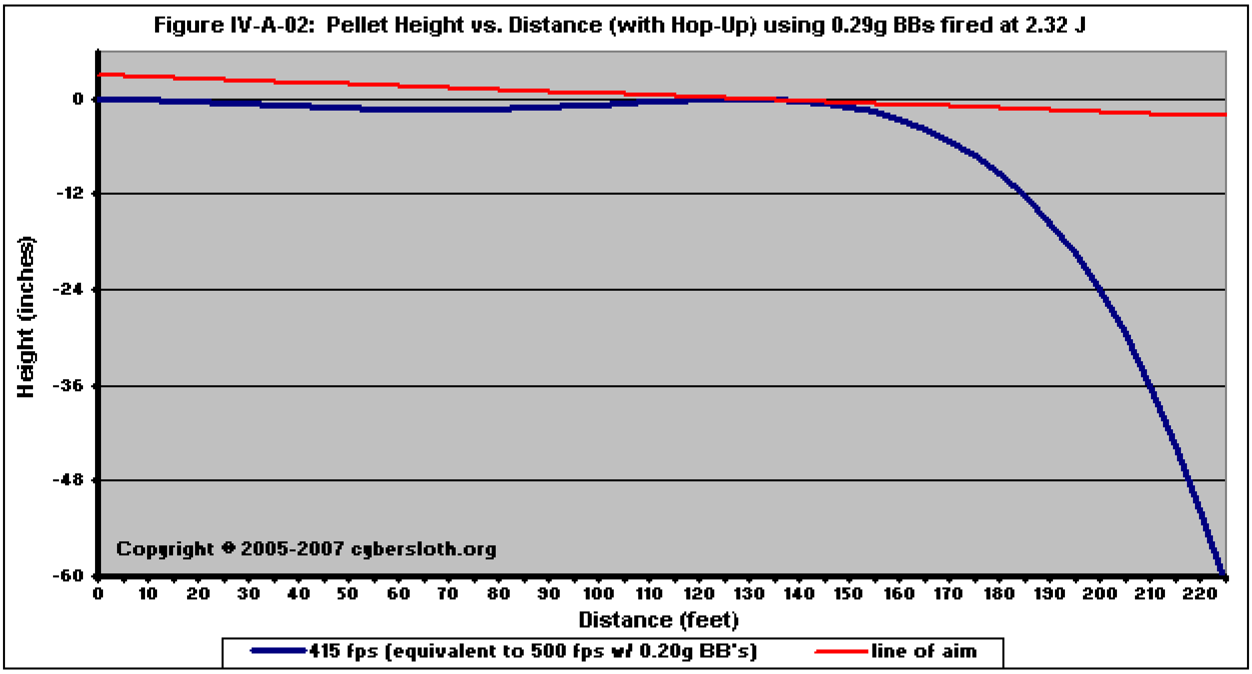
Section IV-A: Definitions of Effective and Maximum Range

I consider the effective range of an airsoft rifle to be the range at which the BB deviates significantly from the shooter's line of aim. Take the example of a stock Tokyo Marui rifle firing at 0.75 fps with 0.25g BB's. Without hop-up, the BB's would quickly begin accelerating downward, consequently deviating from the shooter's line of aim. A quick estimate from looking at Figure IV-A-01, without hop-up the same rifle would be effective out to about 40 feet. With hop-up properly adjusted for the rifle and same ammunition, the effective range increases to around 100 feet, at which point the BB is roughly 6 inches below the line of aim. (Note that the line of aim is not necessarily a height of 0", nor is it perpendicular to the ground. The reason for this is parallax, which can be better understood by visiting a website that goes into detail about marksmanship.)

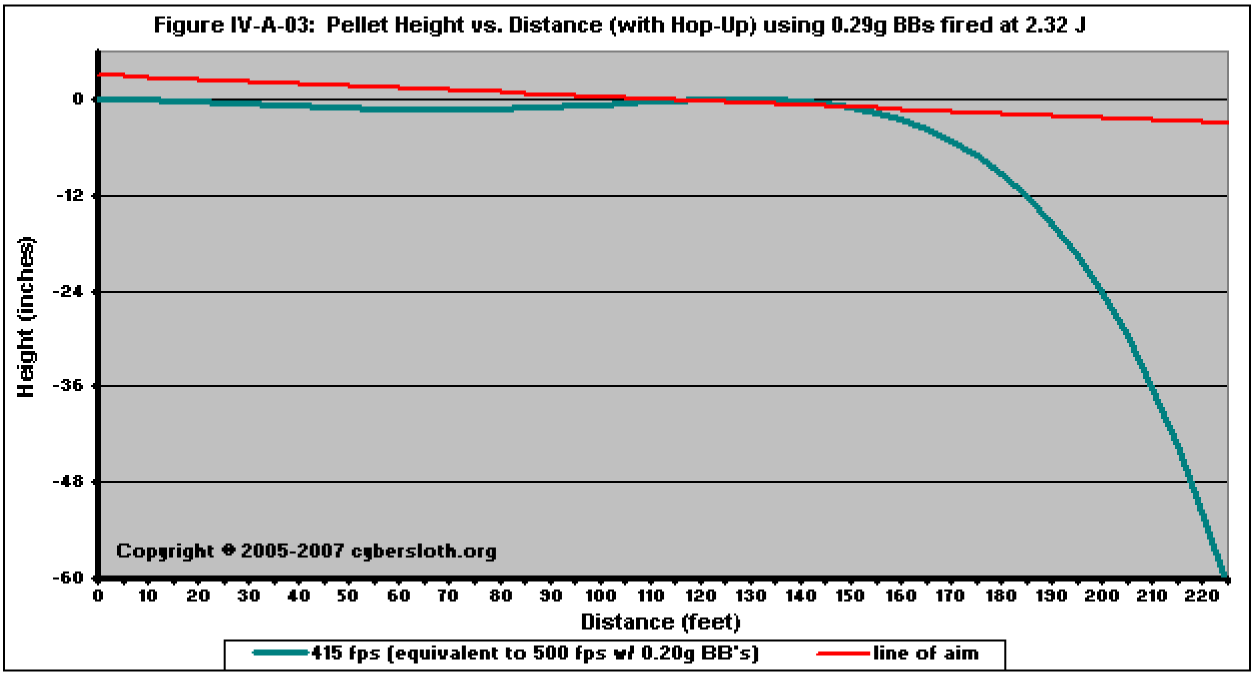


A shooter could sight their rifle in at a different range, thereby increasing the effective range. In Figures IV-A-02, 03, & 04, we can see how this factors in for a sniper rifle firing 0.29g BB's at 415 fps.

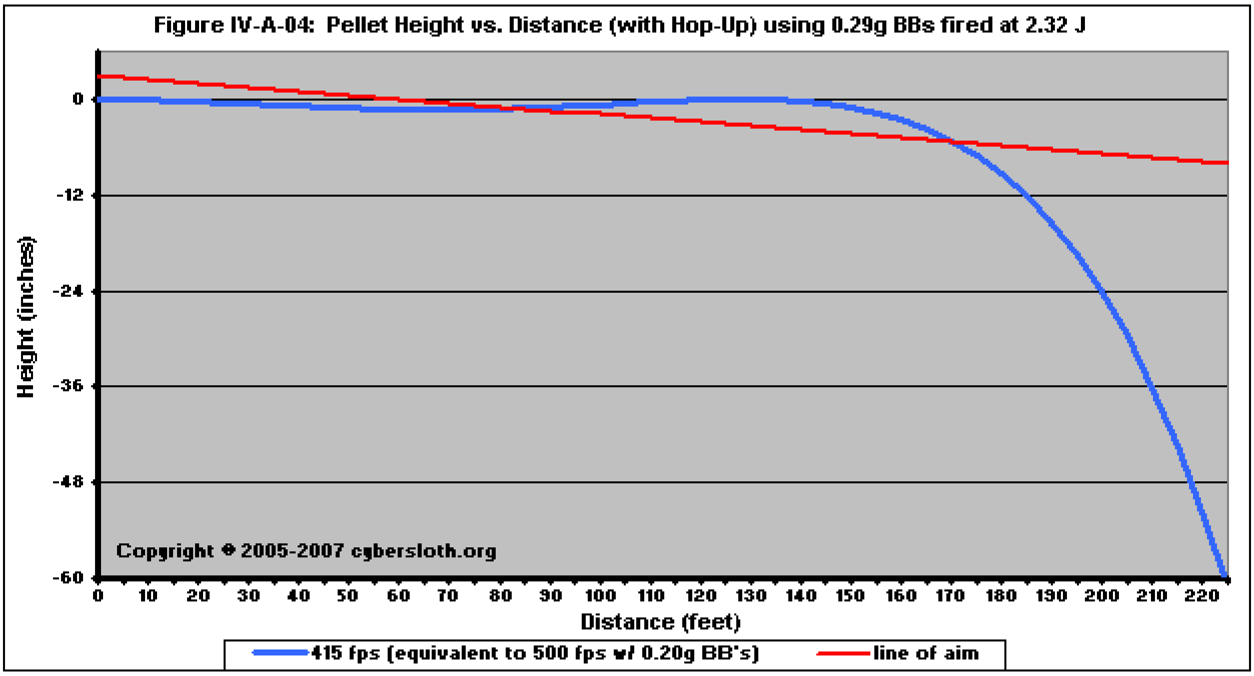
In Figure IV-A-02, the rifle has been sighted in at 130'. In the case, the effective range (taken as the range at which the BB is 6" below the line of aim) is about 170'.



Sighting the rifle to 150' increases the effective range to about 180', as depicted in Figure IV-A-03:

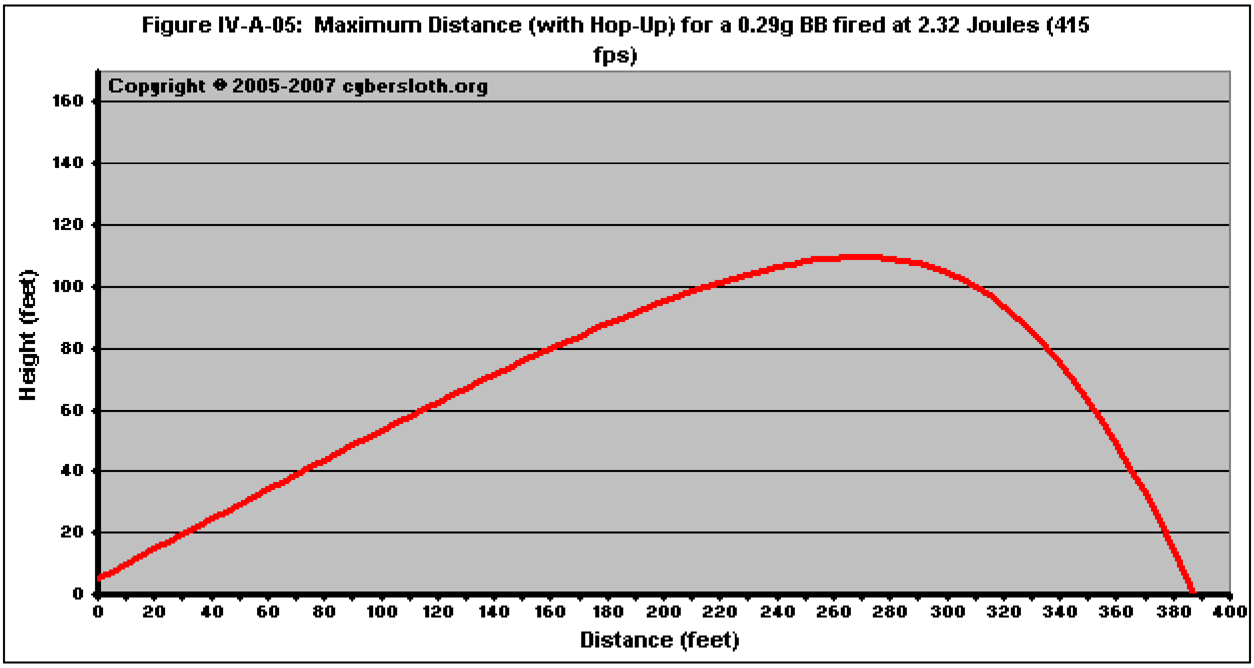


Lastly, sighting the rifle to 170' increases the effective range to about 190', as shown in Figure IV-A-04:



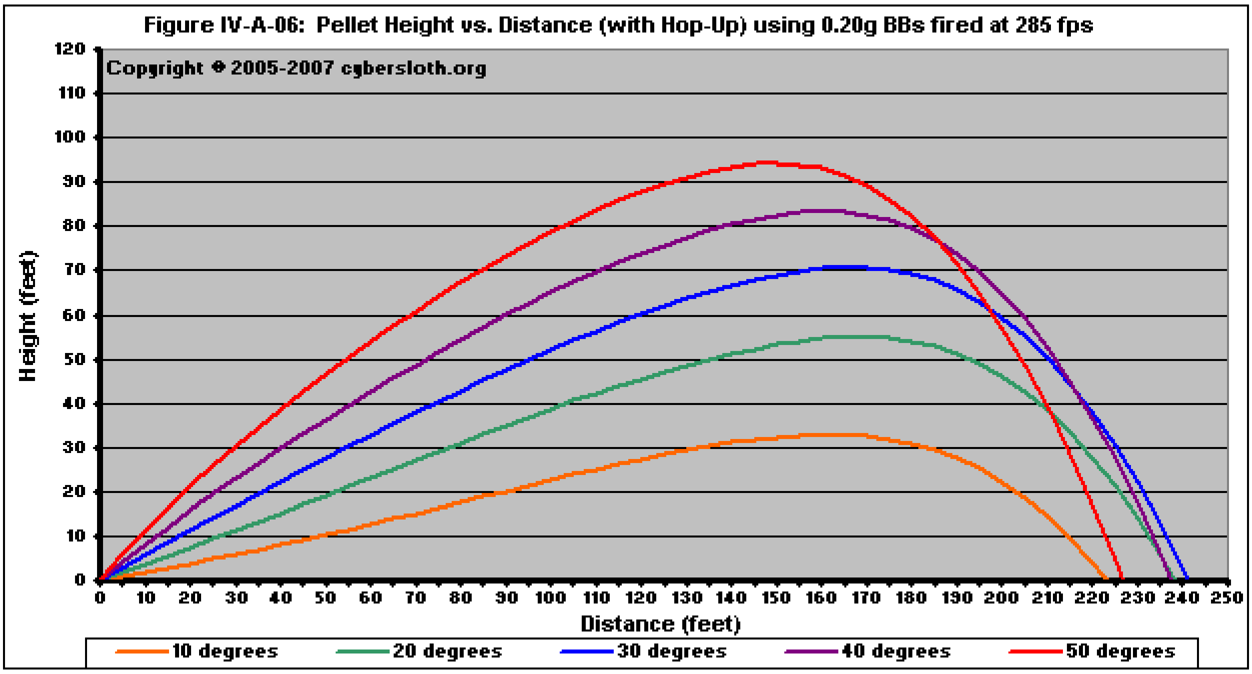
Keep in mind that given the inherent inaccuracy of airsoft, it is very difficult to sight rifles in at long distances. Additionally, it should be obvious that effective range is not strictly governed by muzzle energy nor the weight of the ammunition used. Further, the skills of the shooter come into play in determining the effective range. The effective ranges listed further down are merely rough estimates.

Maximum range was calculated by determining the firing angle that produced the greatest range. While this of no practical use as lobbed shots are in no way accurate, it will give the shooter an idea of how far their BB might travel. Figure IV-A-05 depicts the maximum range of the same 0.29g BB used in the previous example:



At maximum range, the BB is moving very slowly, often at a velocity near its terminal velocity (which is explained in Section 01-A-02: Density / Volume and Terminal Velocity). At such ranges, the BB will impact with very little kinetic energy, striking with about the same intensity as a small raindrop. In the above example, at a range of 385 feet the 0.29g BB will be moving at roughly 49 fps (or about 0.03 J), not even enough to dent a sheet of paper.

The table below shows shows the firing angle necessary to achieve the maximum range. Note that this is not necessarily 45 degrees. In a vacuum, a firing angle of 45 degrees would provide the greatest range. However, given that the BB falls rapidly during the terminal phase of its trajectory, an angle lower than 45 degrees is necessary to achieve maximum range. Figure IV-A-06 shows how trajectories vary by firing angle for a 0.20g BB fired at 0.75 J. .



Section IV-B: Effective Range and Recommended BB Weight

Weight is the recommended BB weight necessary to achieve the listed effective range.

Modified Muzzle Velocity is the muzzle velocity achieved with the recommended weight BB at the given muzzle energy.

Effective Range is the range at which the theoretical BB fired from a properly tuned rifle will experience significant deviation from the *average trajectory*, with a maximum deviation of 6" (explained above). These numbers are not absolute; marksman skills (or lack thereof) will either diminish or extend the range. Further, rifles with exceptional consistency between shots can achieve greater effective ranges, as can a shooter by accounting for holdover. .

Maximum Range is the maximum possible range achievable when using the recommended weight. Keep in mind that a tailwind or higher altitudes / temperatures will allow for a longer ranges.

Maximum Angle is the firing angle that would achieve the maximum range with the listed weight.

MED is the minimum engagement distance for the given velocity and BB weight, allowing for a maximum impact energy of 1.00 J.

Muzzle Energy	Velocity w/ 0.20g BB	Weight (grams)	Modified Muzzle Velocity	Effective Range	Maximum Range	Maximum Firing Angle	MED (feet)
0.37 J	200 fps	0.20	200 fps	65	215 feet	26 degrees	--
0.47	225	0.20	225	75	230	26	--
0.58	250	0.20	250	85	240	26	--
0.70	275	0.25	246	95	265	26	--
0.84	300	0.25	268	105	285	26	--
0.98	325	0.25	291	115	305	26	--
1.14	350	0.25	313	125	315	25	10
1.31	375	0.25	335	135	325	25	15
1.49	400	0.25	358	145	335	25	20
1.68	425	0.30	347	155	355	25	35
1.88	450	0.30	467	160	365	25	40
2.10	475	0.30	389	170	380	25	50
2.32	500	0.30	408	180	390	25	55
2.56	525	0.30	429	185	405	24	60
2.81	550	0.30	449	190	405	24	65

3.07	**575**	**0.36**	**429**	**200**	**455**	**22**	**85**
3.35	**600**	**0.36**	**447**	**210**	**465**	**21**	**95**
3.63	**625**	**0.36**	**475**	**220**	**475**	**20**	**100**
3.93	**650**	**0.43**	**443**	**225**	**530**	**20**	**125**
4.24	**675**	**0.43**	**460**	**230**	**535**	**19**	**135**
4.55	**700**	**0.43**	**477**	**235**	**545**	**19**	**140**
All calculations were made using a standard altitude of sea level and temp of 68 F / 20 C.							

Regarding weight: using the next higher or next lower weight will not significantly alter the effective range. So if you have a 3.35J rifle, you will not see that much of a difference if you are using 0.29g BB's or 0.43g BB's instead of the recommended 0.36g BB's. In fact, most sniper rifles benefit from precision-crafted BB's such as Maruzen SGM's more so than simply increasing the mass of the projectile. Additionally, the lighter 0.29g BB's are safer and require a lesser MED due to greater energy dissipation when compared to the 0.36g and 0.43g BB's.

However, BB weight should be a personal preference rather than simply using what has been listed for posterity's sake. For instance, even though 0.20g BB's are recommended for 200 fps rifles, I still use 0.25g BB's with the AEP I have as they are more consistent for me (and frankly, there is a reason that Tokyo Marui includes 0.25g BB's with their rifles). Ultimately, though, I would still like to stress that these are the calculated recommended weights -- please use what you feel most comfortable using.

Section IV-C: Effective Range Observations

It should be expected that as muzzle energy is increased, effective range increases. However, increasing muzzle energy ultimately succumbs to the law of diminishing returns, meaning that energy increases do not correlate to range increases in a linear fashion. To better explain, consider the effective range gained when going from 200 fps to 300 fps (relative to 0.20g), a gain of some 40 feet (105' - 65'). A linear increase would mean that going from 300 to 400 fps would show another gain of 40 feet effective range, a 400 to 500 fps increase would exhibit yet another gain of 40 feet, so on and so forth. Instead, we see that for every 100 fps gained, we gain less and less effective range. Consider the following chart:

Initial Muzzle Velocity	Upgraded Muzzle Velocity	Original Range	Upgraded Range	Range Gained
200	300	65	105	40
300	400	105	145	40

400	500	145	180	35
500	600	180	210	30
600	700	210	235	25

As is shown, a increase of 100 fps will garner an extra 40' of range up to about 400 fps, and thereafter the gains come in smaller increments. Even so, the decrease in gained range is much less than it would have been if a constant BB weight were used. By increasing the weight of the projectile, the gained range in going from 600 fps to 700 fps is still 25 feet, not too far reduced from the 40 feet gains observed when upgrading from lesser velocities. If 0.20g BB's were the only available weight, the above chart would look more like this:

Initial Muzzle Velocity	Upgraded Muzzle Velocity	Original Range	Upgraded Range	Range Gained
200	300	65	105	40
300	400	105	140	35
400	500	140	170	30
500	600	170	195	25
600	700	195	210	15

The above chart demonstrates the diminished returns much more clearly. Of course, the chart is somewhat misleading as 0.20g BB's exhibit atypical behavior at velocities above 500 fps due to peculiarities of Magnus Lift. As of such, it would be impossible to achieve the ranges listed because the 0.20g BB's would not provide any degree of long-range accuracy with high-energy rifles.

As an aside, this trajectory project suggests that both the size and weight of airsoft projectiles are optimal for hop-up. Many shooters -- particularly snipers -- wonder why SGM's only weigh in at 0.29g. Based on the data presented in this study, BB's in the range of 0.28g to 0.30g seem to offer good trajectory characteristics while ensuring safety due to the lesser weight used. For instance, the difference in range of a 0.30g BB and a 0.43g BB at 3.93 J is negligible when considering the 6" envelope used to determine effective range. Granted, a shooter using holdover to snipe more distant targets may see an extra 20 feet of effective range at such a high muzzle energy, but for most practical sniper rifles, 0.43g BB's (even if they were precision crafted) appear to offer no significant advantage over the lighter 0.29g BB's. Given the higher costs associated with producing heavier BB's, and the already high cost of precision crafted BB's, 0.29g BB's seem to be a good compromise.

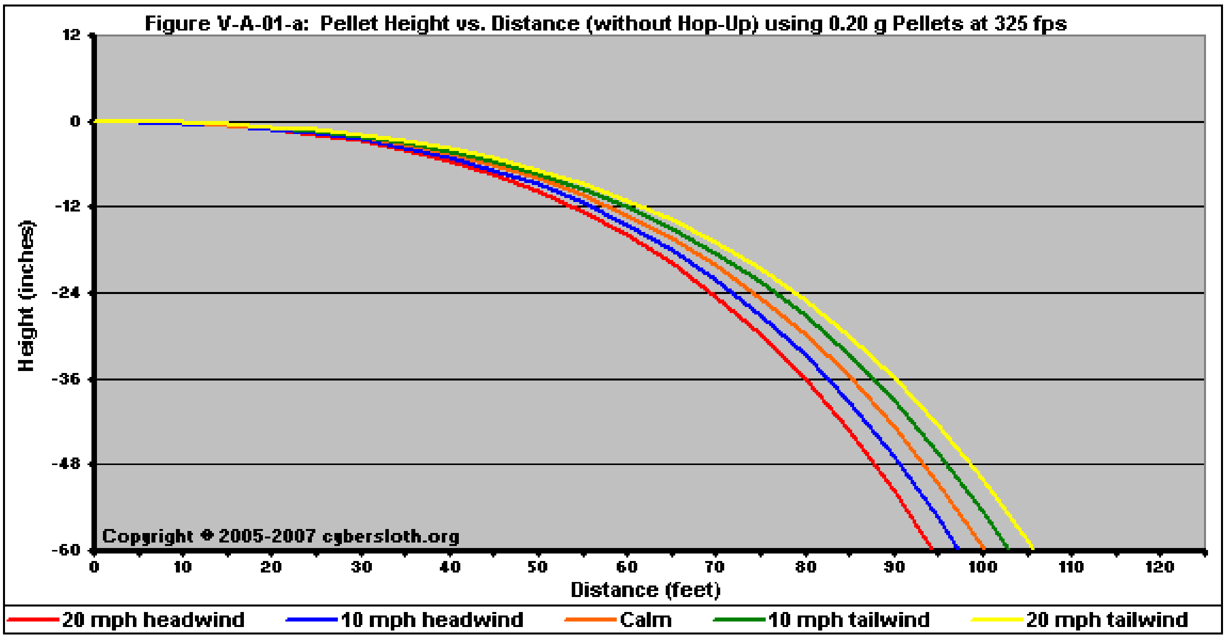
Additionally, one last point I'd like to stress is that the correlation between effective range and muzzle energy is not absolute. It is far better to have a rifle that is able to achieve greater consistency between shots rather than a rifle that is very powerful, as the former will often provide greater effective range.

The Airsoft Trajectory Project

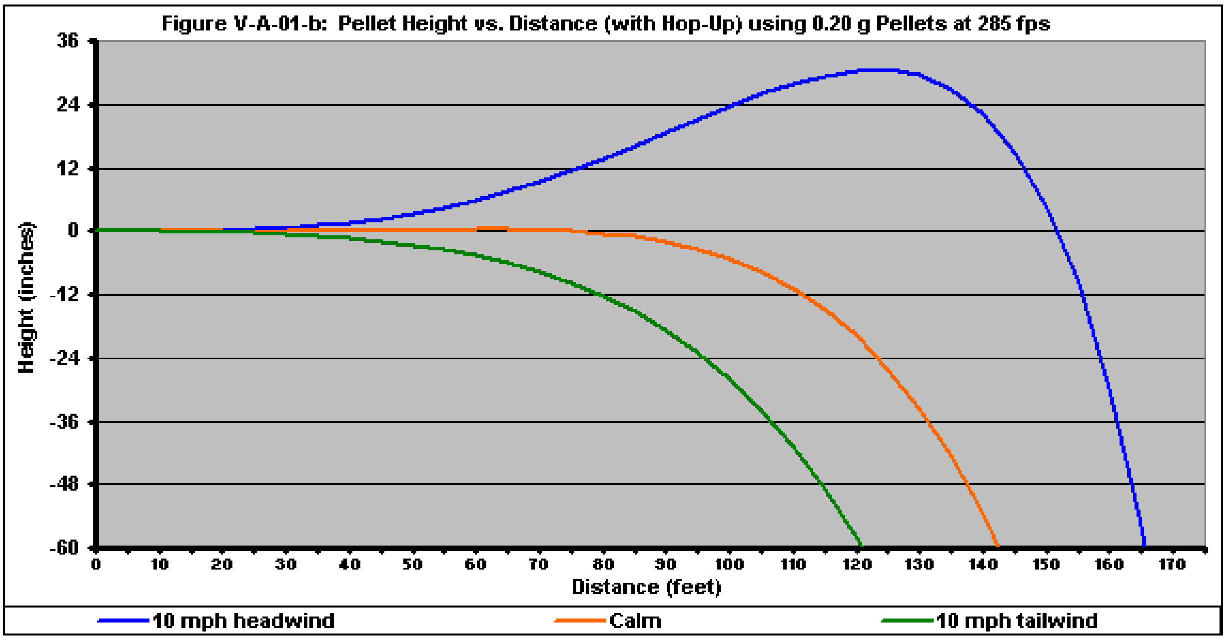
ATP - Headwind/Tailwind Component

Section V-A-01: Headwind/Tailwind Component

When firing into a headwind (wind flowing from target to shooter) or tailwind (wind flowing from shooter to target), it is perfectly sensible to expect the wind to affect the trajectory of the BB. Figure V-A-01-a shows the modeled trajectories of a 0.20g BB fired at 325 fps into varying wind conditions. As expected, a non-spinning shot fired into a headwind would experience additional drag, consequently retarding its forward motion. Conversely, a tailwind will prolong the BB's trajectory as the BB will experience less drag over the course of its trajectory.



While the trajectory of a BB without hop-up seems intuitive, trajectory of a BB with hop-up applied seems to defy our expectations. Figure V-A-01-b shows the trajectories of BB's with hop-up applied for a 0.85 J rifle fired into the wind:



Common sense tells us that a headwind should produce lesser trajectories. Unfortunately, this isn't what the model predicted. Initially, there was worry that there was something wrong with the model and testing was in order. Using a 1.5 J rifle (400 fps w/ 0.20g BB's), we set out to see how headwinds and tailwinds affected trajectory. First, the direction of the wind was determined and two targets were set up on a flat field, 150' apart with the two targets aligned with the direction of wind. The next step of the test was to take the rifle to the downwind target and fire shots at the upwind target (i.e., into the wind). Then, shots were fired from the upwind target to the downwind target. Strangely, observation matched the model results -- shots fired into the wind went further than the shots fired with the wind at our backs.

Some quick data mining helped to explain why spinning shots gained distance when fired into a headwind. As was explained in detail in Section III: Effects of Hop-Up, the amount of lift generated by a spinning BB is directly dependent upon the ratio of rotational velocity (V) to the linear velocity (U). High V/U causes significant lift; low V/U does not. When firing into a headwind, the linear velocity U decreases at a much faster rate than normal, while the rotational velocity V decreases at the same rate regardless of whether it is calm, a headwind or tailwind. For a headwind, V stays the same but U is lower, meaning that the ratio of V/U is ultimately higher, leading to greater lift. Essentially, shots fired into the wind tend to "over hop" leading to greater trajectory.

Another, perhaps simpler way to look at it is to realize that trajectories for normal projectiles are greatly limited by air resistance. A headwind will provide greater air resistance for normal projectiles, limiting their trajectory. Spinning BB's are primarily limited by the effects of lift. A headwind will ultimately lead to greater lift at range, thereby increasing the trajectory of spinning BB's.

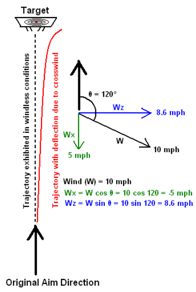
ATP - Crosswind Component

Section V-A-02: Crosswind Component

The crosswind component is taken as the wind component perpendicular to the line drawn from rifle to target. Just as was done with the headwind / tailwind component, the crosswind has to be reduced from the total wind component into a vector. This is done by taking the wind direction in relation to the original aim direction.

Figure V-A-02-a illustrates how to divide the wind direction into components. In this case, the wind is blowing at ten miles per hour at an angle of 120º to the direction aimed. This means that the BB will experience a headwind of five miles per hour and a crosswind of 8.6 miles per hour. For calculating the deflection due to crosswind, we need only worry about the crosswind component, or 8.6 mph.

Figure V-A-02-a: Wind Component Diagram



The next problem is determining how much wind affects the trajectory. This can easily be calculated by using the drag equation in Section I-D-01. The amount of deflection will be determined by the lateral velocity of the BB. In turn, the lateral velocity is dependent upon both how long the BB has been subjected to the crosswind and how fast the crosswind is moving in relation to the BB (or for how long and much of an acceleration the BB experiences..

Most people tend to adhere to either of two theories as to why a BB will or will not be affected by crosswind. The first theory is that the heavier BB's will retain their inertia better, thereby being less affected by a crosswind. As of such, the heavier BB will experience a lesser degree of deflection downrange.

The second theory is that all BB's, regardless of weight, will be "caught" by the wind and will be moving at roughly the same lateral speed as that of the medium that the BB's are passing through.

Most lay target shooters adopt this latter argument. This is understandable as it makes calculating wind deflection very easy. Additionally, most target shooters are dealing with a very dense projectile fired at a very high velocity.

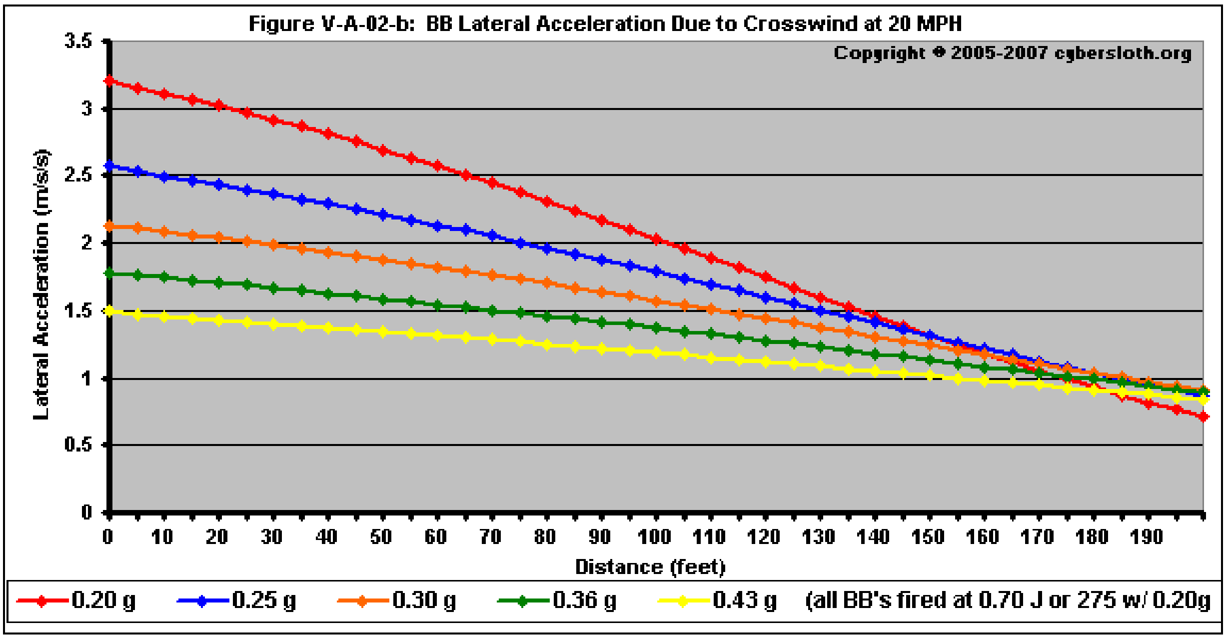
Unfortunately, this latter theory is incorrect. Firearm and 0.177" BB shooters can get away with this to a slight degree because, again, they are dealing with a dense projectile moving at a very high velocity. For both bullets and steel BB's, the projectile reaches the target relatively quickly. Furthermore, the increased density (and mass) of both types of projectiles dictate that the force necessary to accelerate the projectile laterally must be significantly higher.

This IS NOT the case for airsoft shooters, however, as the projectile has a density much less than that of a bullet (or even a 0.177" steel BB) and the muzzle velocities tend to be considerably less for airsoft rifles when compared to the muzzle velocities of firearms and 0.177" BB guns. To illustrate this point, let's consider the following example:

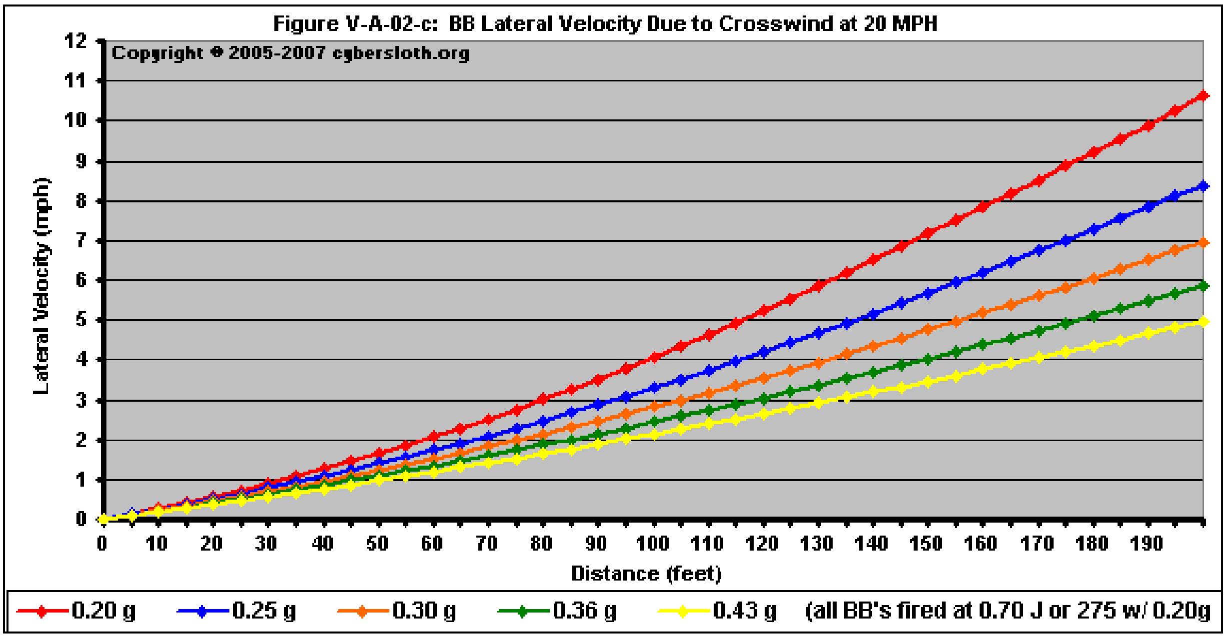
Imagine that the shooter is firing at a target that is 100 feet downrange, and that he is firing a 0.20g BB at 275 fps into a a 20 mph crosswind. If the shooter estimates that the *average* velocity of the BB will be 160 feet per second (which is actually correct), then they can readily expect that it will take about 0.70 seconds for the BB to reach the target (again, the calculations reflect reality). If the BB maintains a constant lateral velocity of 20 mph (or about 30 feet per second) from the moment it leaves the barrel, then it would be deflected 21 feet to the side by the time it has traveled 100 feet. 21 feet is an awful lot -- I think that most airsoft shooters doing tests in such a crosswind would quickly determine that the lateral deflection is significantly less than 21 feet. In reality, the BB would *only* be deflected about 22 inches, a far cry from the 21 feet estimated earlier. Why the huge discrepancy? As will be illustrated below, the BB doesn't move at 20 mph upon exiting the barrel -- in fact, it doesn't even get close to 20 mph.

Figures V-A-02-b through V-A-02-e depict the lateral characteristics of different BB's that have been fired at 0.70 Joules (or the equivalent of 275 fps with a 0.20 g BB) into a 20 mph crosswind.

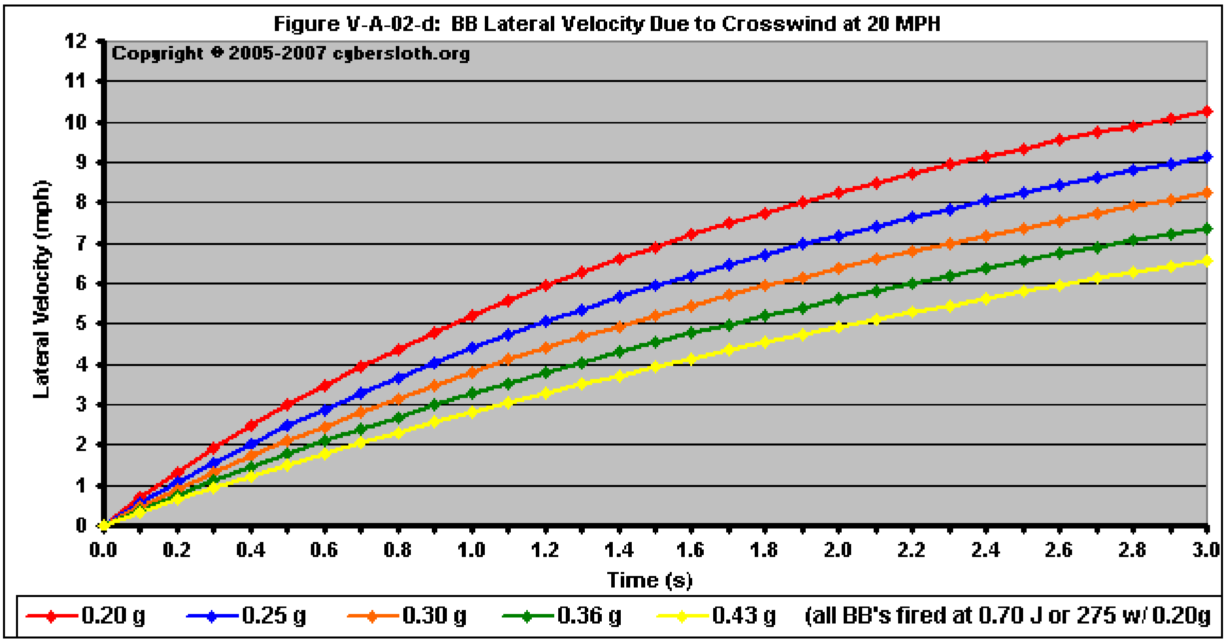
Drag acceleration is a function of mass, and as of such we would expect that BB's with a higher mass will experience a lesser acceleration. Figure V-A-02-b depicts lateral acceleration of a variety of BB masses. Consequently we see that the 0.20 gram BB's undergo a more pronounced acceleration due to the crosswind. Also notice that the acceleration decreases with time and distance. The reason that the acceleration decreases with time is that the BB is experiencing less drag with the passage of time. Think of it this way: when the BB leaves the barrel, it is subjected to drag that is a component of the wind velocity in relation to the BB. Immediately after leaving the barrel the BB is experiencing drag due to a lateral wind speed of 20 mph (or 8.94 m/s or 29.33 fps). As time passes, the BB begins to become a part of the moving medium. As this happens, the lateral wind speed in relation to the BB decreases, thereby reducing the lateral drag force.



In Figure V-A-02-c, we see the lateral velocity of the BB in miles per hour. Initially, all BB's have a lateral velocity of 0 mph. As time passes, the BB's gain velocity as they accelerate laterally. Notice that at 100 feet, the 0.20 gram BB will be moving laterally at about 4.1 mph. By contrast, notice that the 0.43 gram BB is moving laterally at about 2.2 mph. It is easy to see why heavier BB's fare better in higher winds than lighter BB's.



For the velocity graphic, I decided to show the lateral velocities out to a distance of 200 feet. The reason I did this was to illustrate that even the relatively light 0.20 gram BB will not reach a velocity of 20 mph at a distance of 200 feet. In truth, the projectile will never reach 20 mph as the lateral acceleration decreases steadily due to a continuously diminishing drag force. This might seem to contradict the fact that the the lateral acceleration is diminishing with time, however it only appears this way because the previous plot was generated as velocity versus distance. In a plot of velocity versus time, we can clearly see that the lateral velocity is decreasing, as is depicted in Figure V-A-02-d.

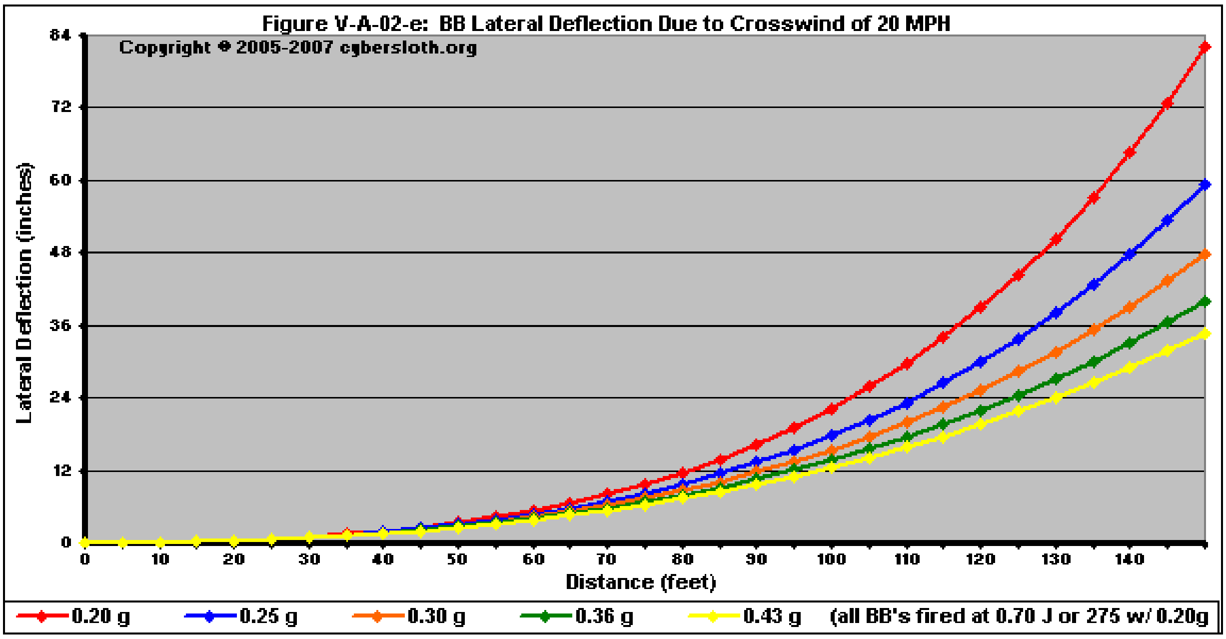


For the velocity vs time analysis, I decided to show the lateral velocities for a long model run of 3.0 seconds. This is completely academic, though, as it is impossible for a horizontally-aimed BB to stay airborne for three seconds. However, it does illustrate the fact that the projectile will not reach 20 mph after a very long flight time. Just for the sake of argument, I performed an extremely long model run out to 25 seconds. Here are the results:

Time (seconds)	Velocity (m/s)	Velocity (mph)
0	0.00	00.0
5	5.54	12.4
10	6.84	15.3
15	7.42	16.6
20	7.75	17.3
25	7.96	17.8

Again as the projectile's lateral velocity increases, the velocity of the wind in relation to the projectile decreases, reducing the amount of drag experienced by the projectile. Another model run that made calculations out to 50 seconds showed that the BB was still below 19 mph @ 50 seconds.

Finally in Figure V-A-02-e, we can see the actual deflection of the BB's. Even 0.25 gram BB's show a marked reduction in lateral deflection when compared to that of the 0.20 gram BB's. However, increasing the mass of the BB's became a law of diminishing returns. The difference in lateral deflection between a 0.30 gram BB and that of a 0.43 gram BB is only about four inches at a distance of 100 feet. And this is when fired at 275 fps. As we'll see, at higher velocities the differences between the masses becomes less at shorter distances.



Figures V-A-02-f through V-A-02-i deal with projectiles fired at 0.98 Joules (or 325 fps with a 0.20g BB)

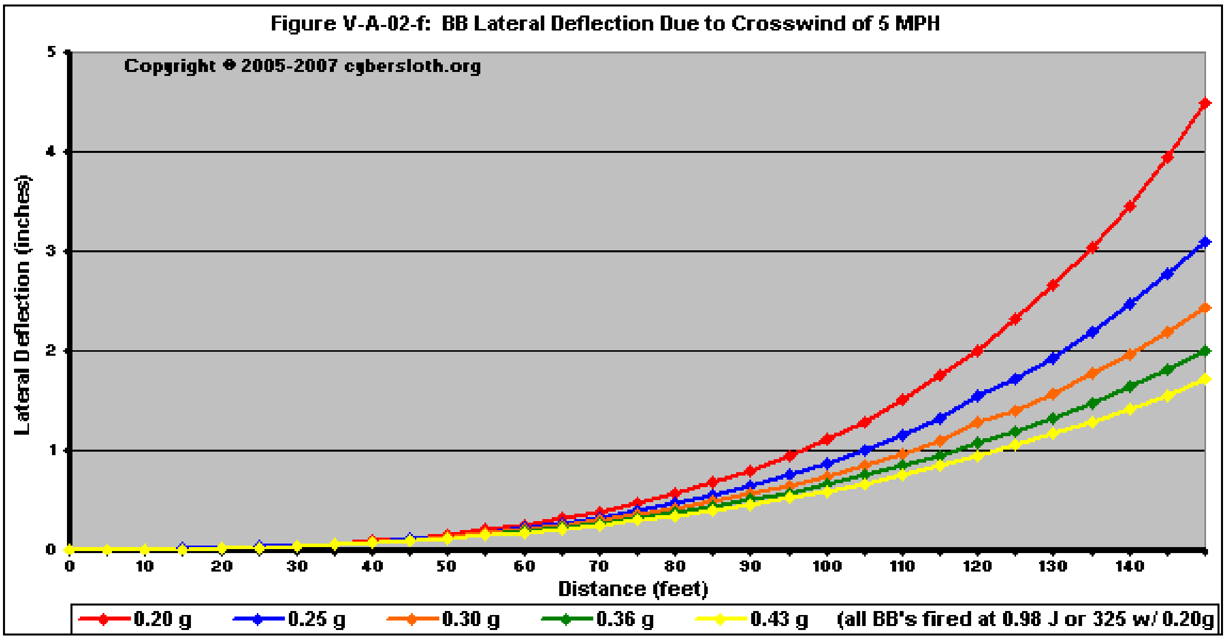


Figure V-A-02-f: BB Lateral Deflection Due to Crosswind of 5 MPH
Copyright © 2005-2007 cybersloth.org
Lateral Deflection (inches)
Distance (feet)
0.20 g
0.25 g
0.30 g
0.36 g
0.43 g
(all BB's fired at 0.98 J or 325 w/ 0.20g

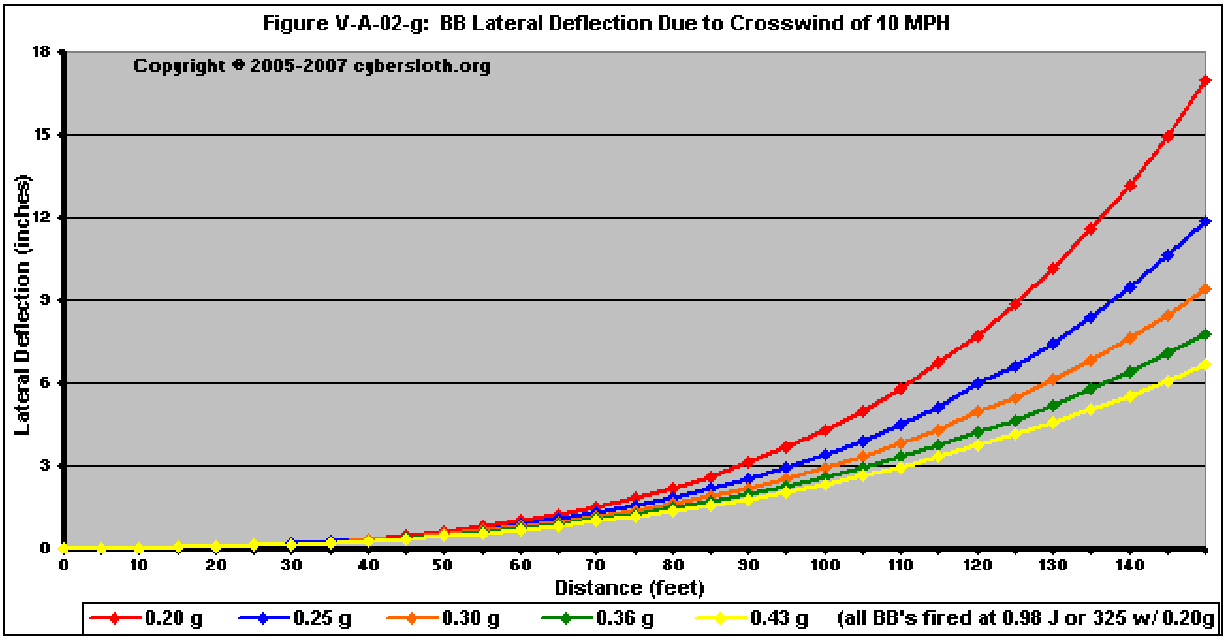


Figure V-A-02-g: BB Lateral Deflection Due to Crosswind of 10 MPH
Copyright © 2005-2007 cybersloth.org
Lateral Deflection (inches)
Distance (feet)
0.20 g
0.25 g
0.30 g
0.36 g
0.43 g
(all BB's fired at 0.98 J or 325 w/ 0.20g

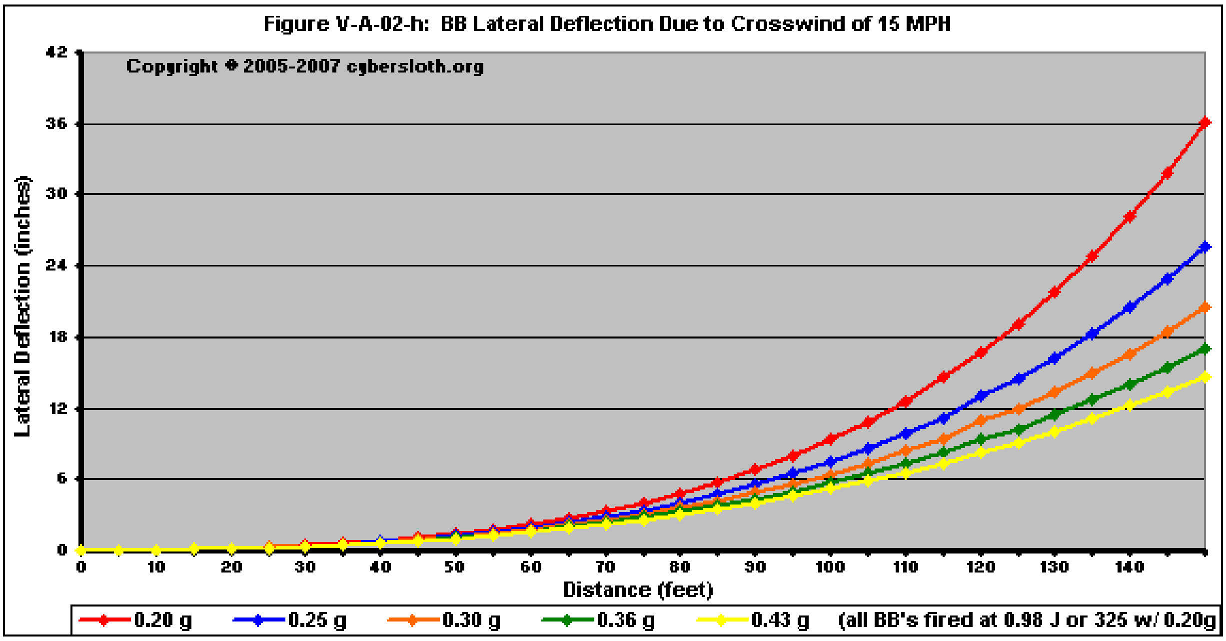
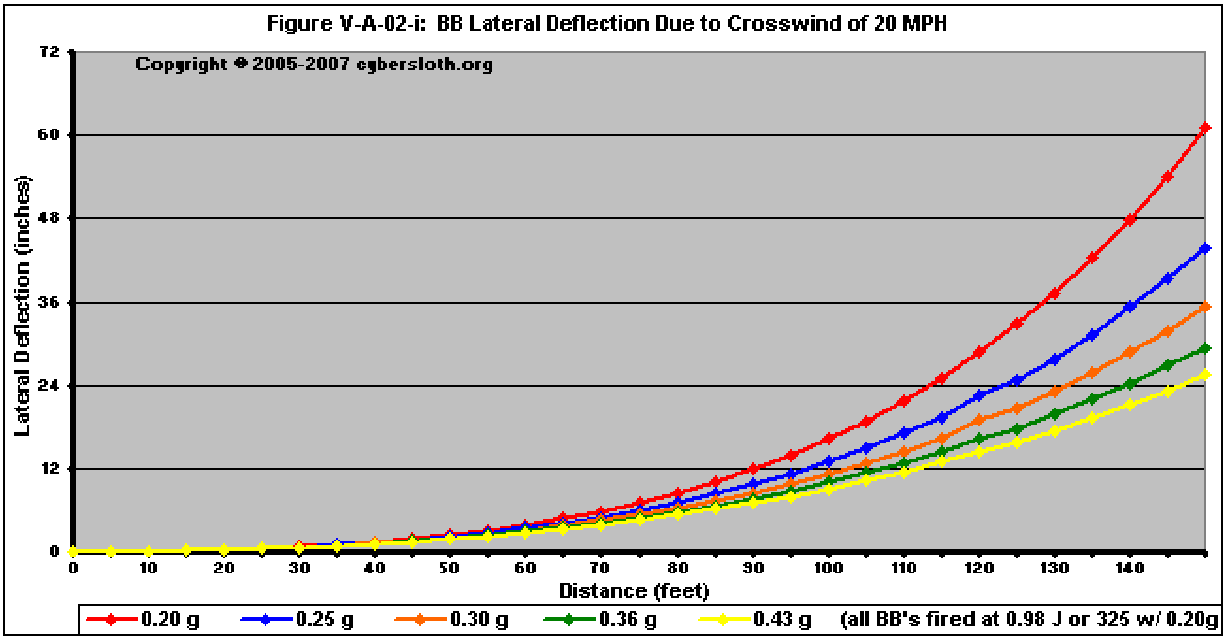
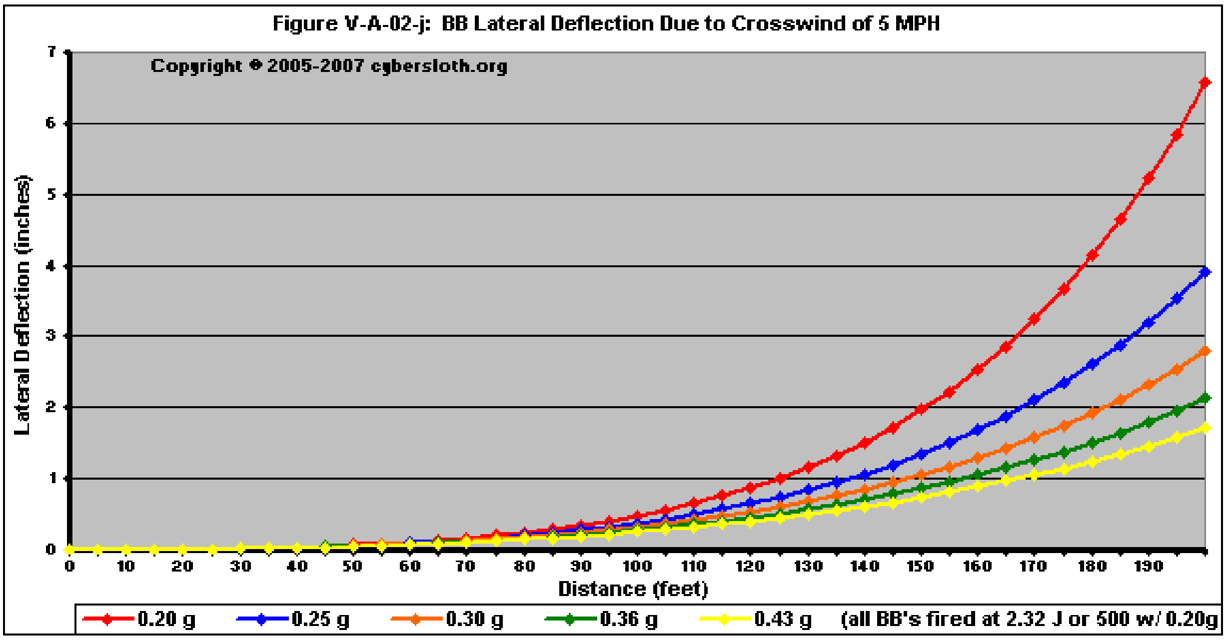


Figure V-A-02-h: BB Lateral Deflection Due to Crosswind of 15 MPH
Copyright © 2005-2007 cybersloth.org
Lateral Deflection (inches)
Distance (feet)
0.20 g
0.25 g
0.30 g
0.36 g
0.43 g
(all BB's fired at 0.98 J or 325 w/ 0.20g



Figures V-A-02-j through V-A-02-m deal with projectiles fired at 2.32 Joules (or 500 fps with a 0.20g BB)



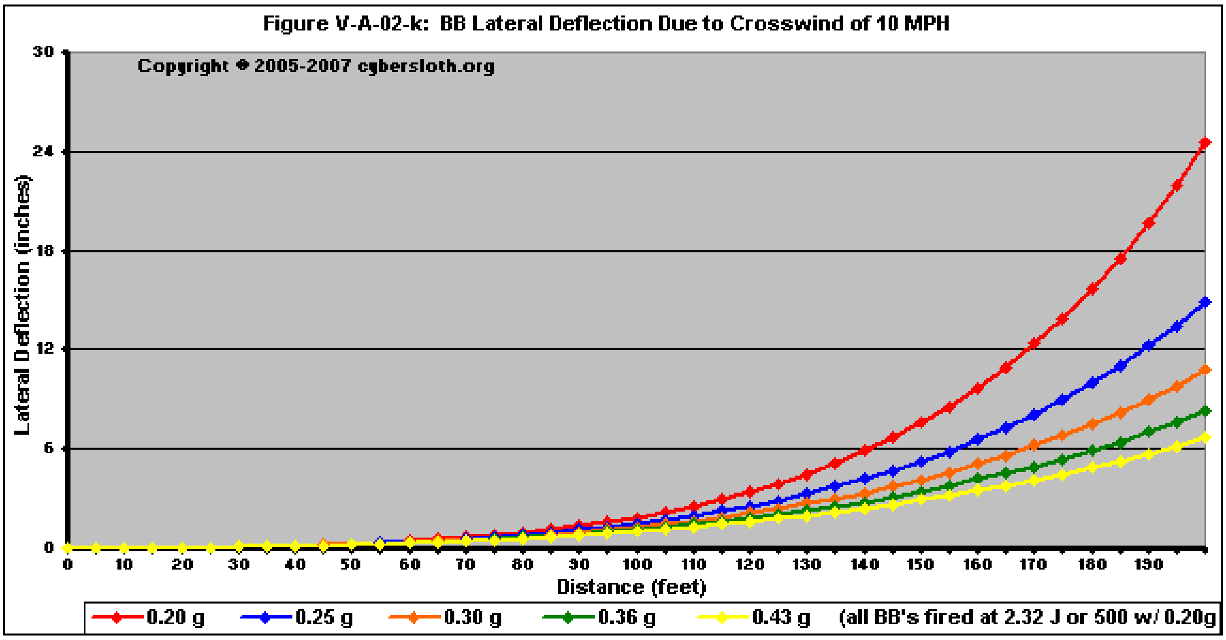


Figure V-A-02-k: BB Lateral Deflection Due to Crosswind of 10 MPH
Copyright © 2005-2007 cybersloth.org
Lateral Deflection (inches)
Distance (feet)
0.20 g
0.25 g
0.30 g
0.36 g
0.43 g
(all BB's fired at 2.32 J or 500 w/ 0.20g

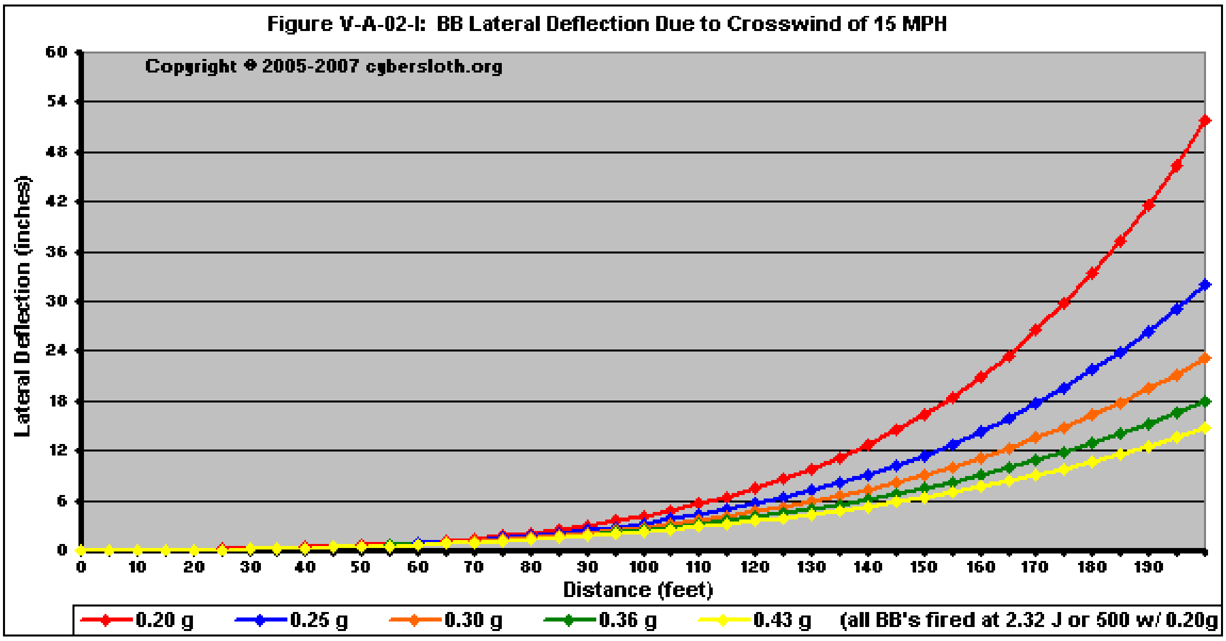


Figure V-A-02-l: BB Lateral Deflection Due to Crosswind of 15 MPH
Copyright © 2005-2007 cybersloth.org
Lateral Deflection (inches)
Distance (feet)
0.20 g
0.25 g
0.30 g
0.36 g
0.43 g
(all BB's fired at 2.32 J or 500 w/ 0.20g

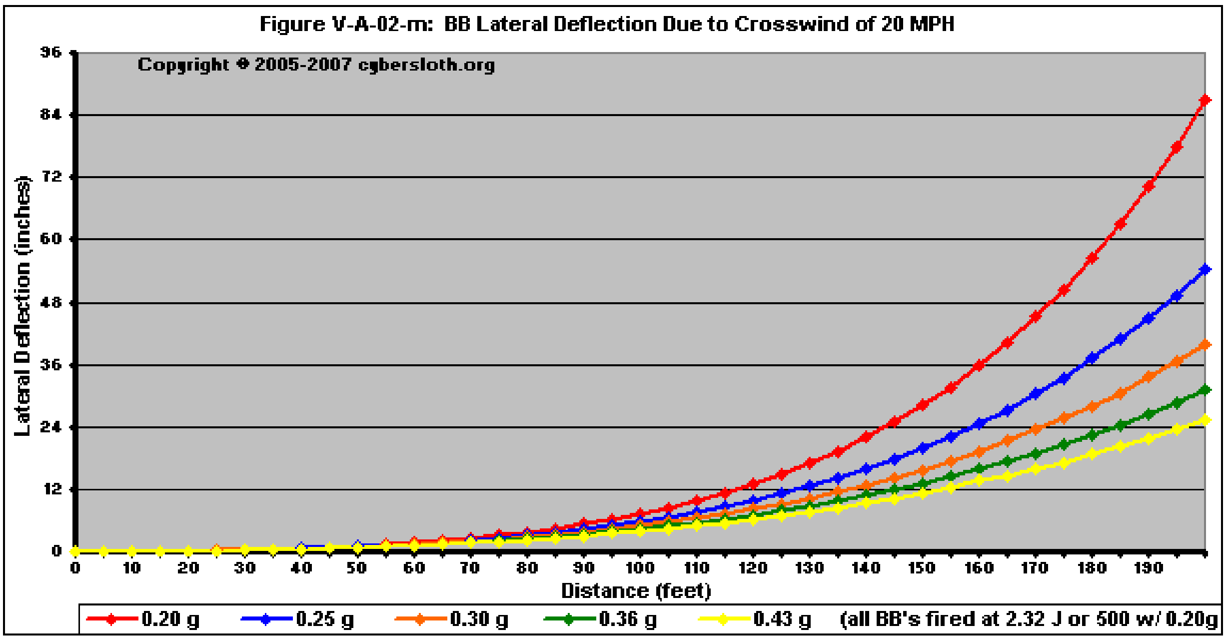
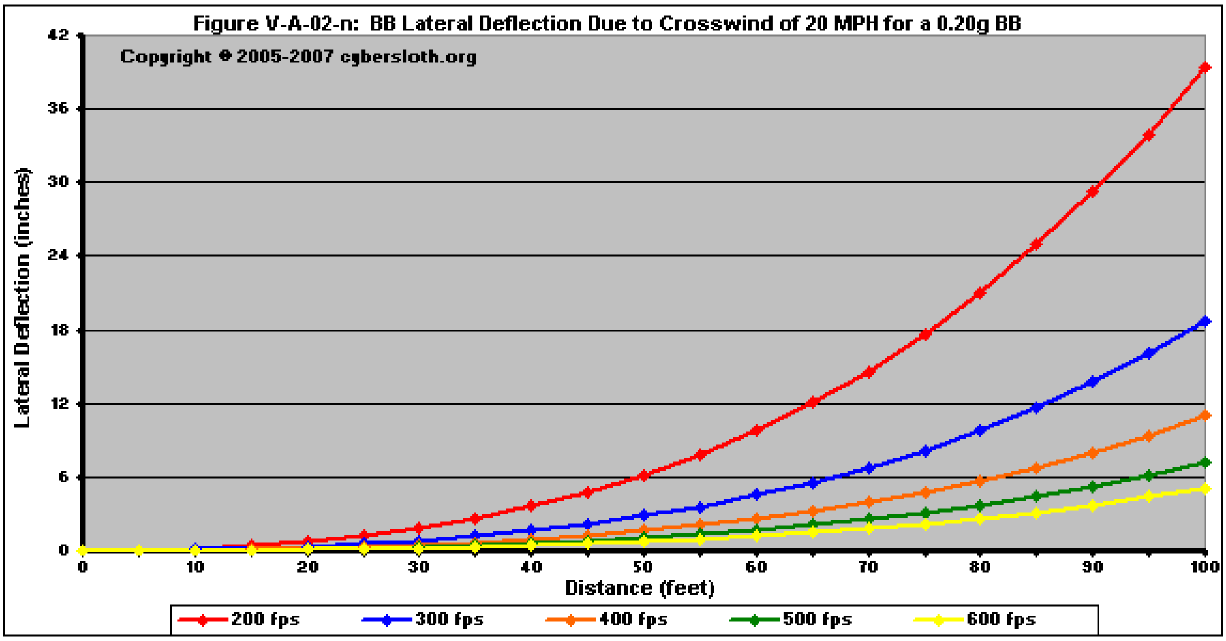


Figure V-A-02-m: BB Lateral Deflection Due to Crosswind of 20 MPH
Copyright © 2005-2007 cybersloth.org
Lateral Deflection (inches)
Distance (feet)
0.20 g
0.25 g
0.30 g
0.36 g
0.43 g
(all BB's fired at 2.32 J or 500 w/ 0.20g

Lastly, Figure V-A-02-n shows how the relationship between muzzle velocity and crosswind. We can see that for equal BB weights, a high-power shot will reach its target having experienced less deflection. For instance, if you were firing at a target 100 feet away with a 20 mph crosswind, a 0.20g BB fired at 400 fps will have only been deflected about 11 inches, whereas a 300 fps shot would have been deflected around 19 inches.

Keep in mind that for an equal time period, the BB moves the same distance laterally for each shot. The high-power shots do not resist crosswind any better than the low-power shots; rather, the high-power shots get further down range in less time. To explain it further, the 600 fps shot reaches 100 feet after 0.32 seconds, after which time it has been deflected about 5 inches. After 0.32 seconds, the 500 fps shot has only gone 90 feet, where it has an equal deflection of 5 inches. By contrast, the 200 fps shot only reaches about 47 feet over the course of 0.32 seconds.



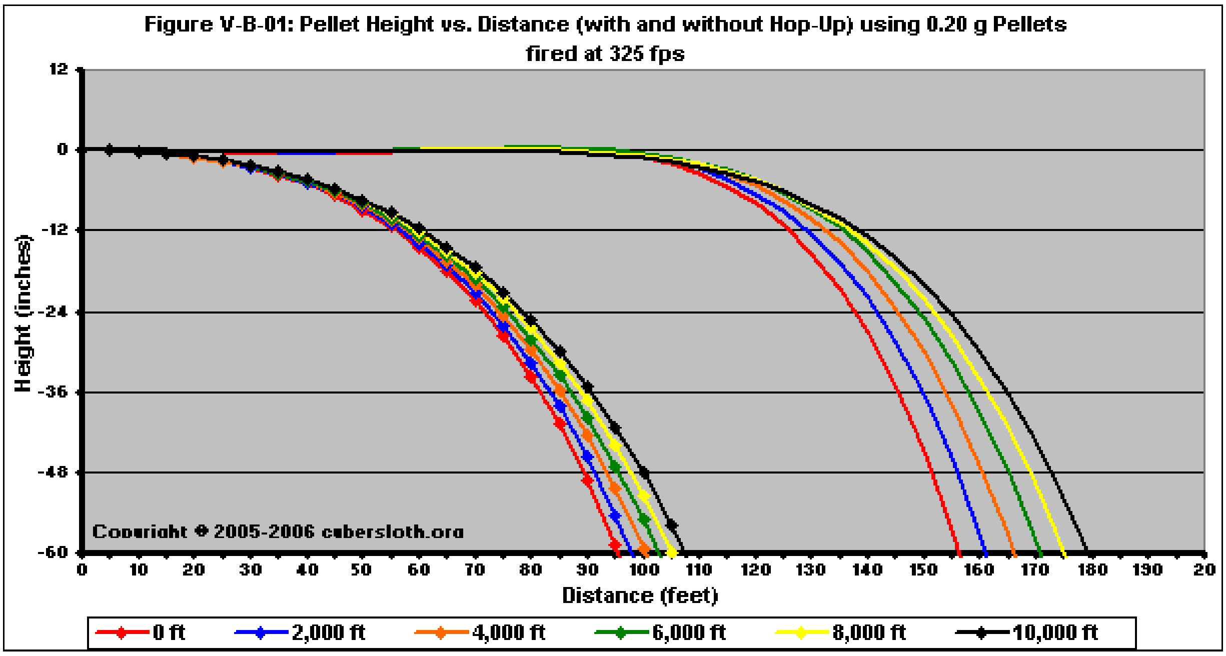
Effect of Altitude on Trajectory

Section V-B: Effect of Altitude on Trajectory

Typically, lower air density -- as is the case in higher altitudes or warmer temperature -- benefits ballistic trajectories. However, this is not straightforward with airsoft due to the effects of hop-up. It is easy to understand how the lower air density allows for a longer trajectory for non-rotating BB's. Initially, I estimated that lower density air would benefit non-spinning BB's (as was fairly obvious), as I had assumed that less air would translate to less lift down range. While it is true that there is less air for a spinning BB to use for generating lift, I neglected to take into account other factors that affect trajectory down range.

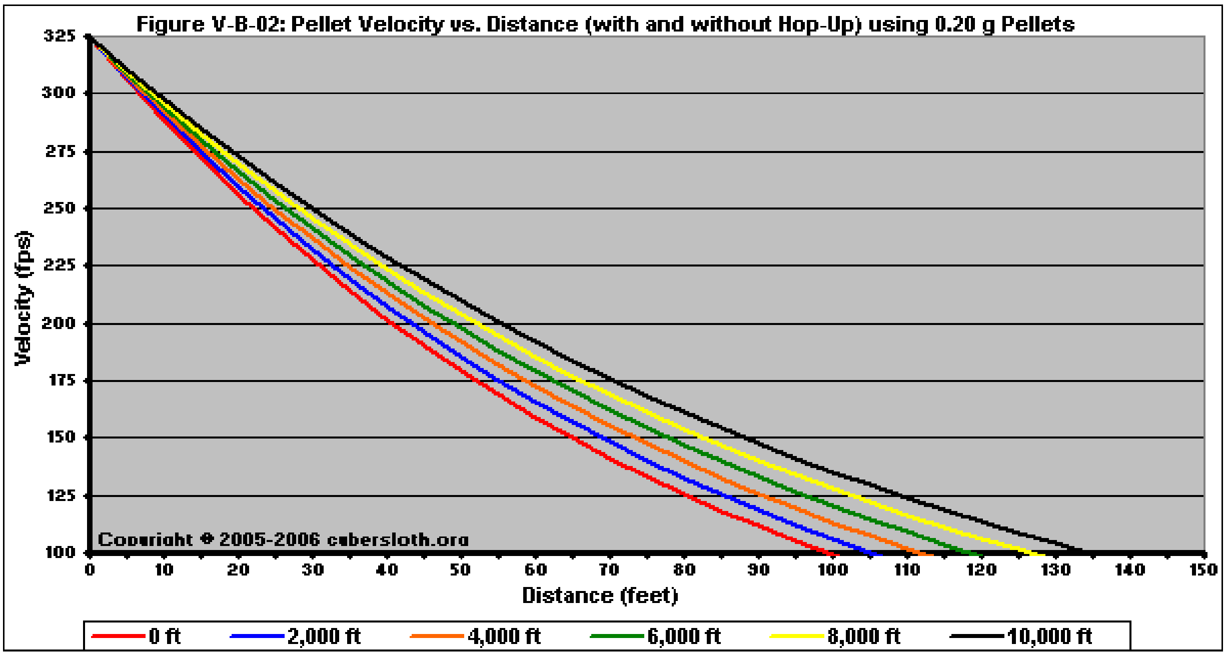
In the case of hop-up, lower air density means that the BB cannot generate as much Magnus Lift. Think of it this way: Magnus Lift comes about due to a momentum shift in the boundary layer (the air immediately surrounding the BB). As the air is less dense, there is less mass for the momentum shift. While this serves as a detriment to the trajectory of spinning BB's at higher altitudes, lower air density also reduces the rate at which spin decay occurs for the rotating BB. This effect means that the BB will be spinning a significantly higher speed down range. The higher rate of spin down range serves to negate the effects of the lack of air mass needed for Magnus Lift. In other words, in less-dense air the BB is able to achieve more lift even though there is less medium to work with.

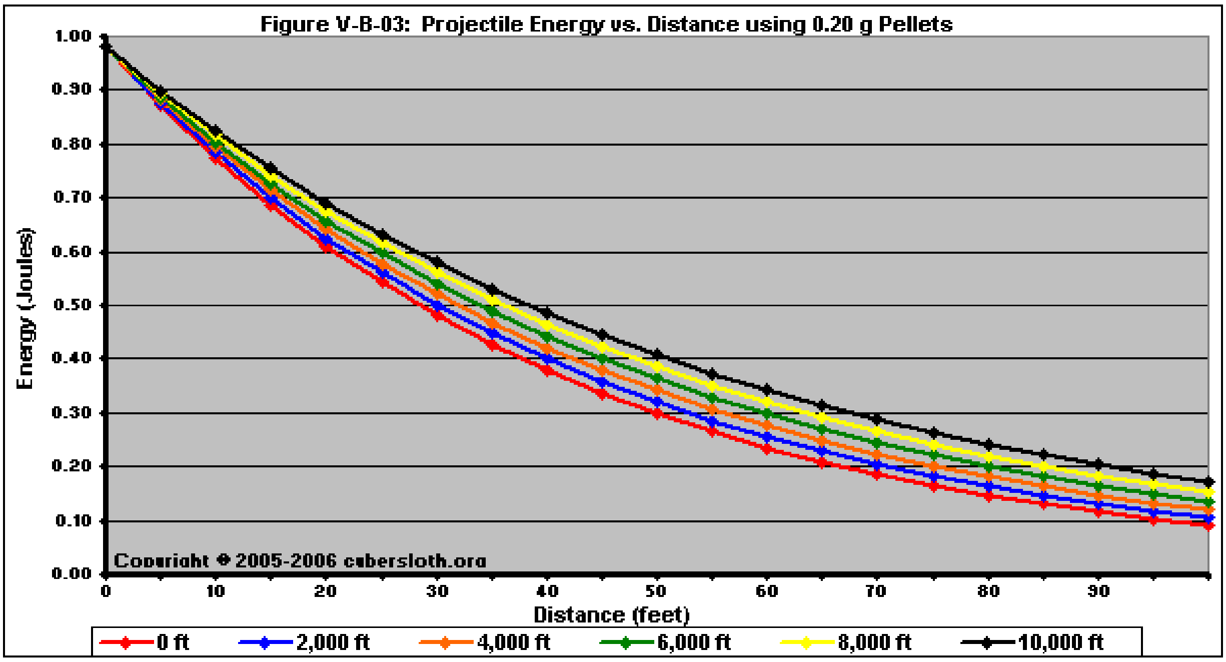
In Figure V-B-01, notice that the higher altitude BB's outrange those at lower altitudes, both with and without hop-up applied.



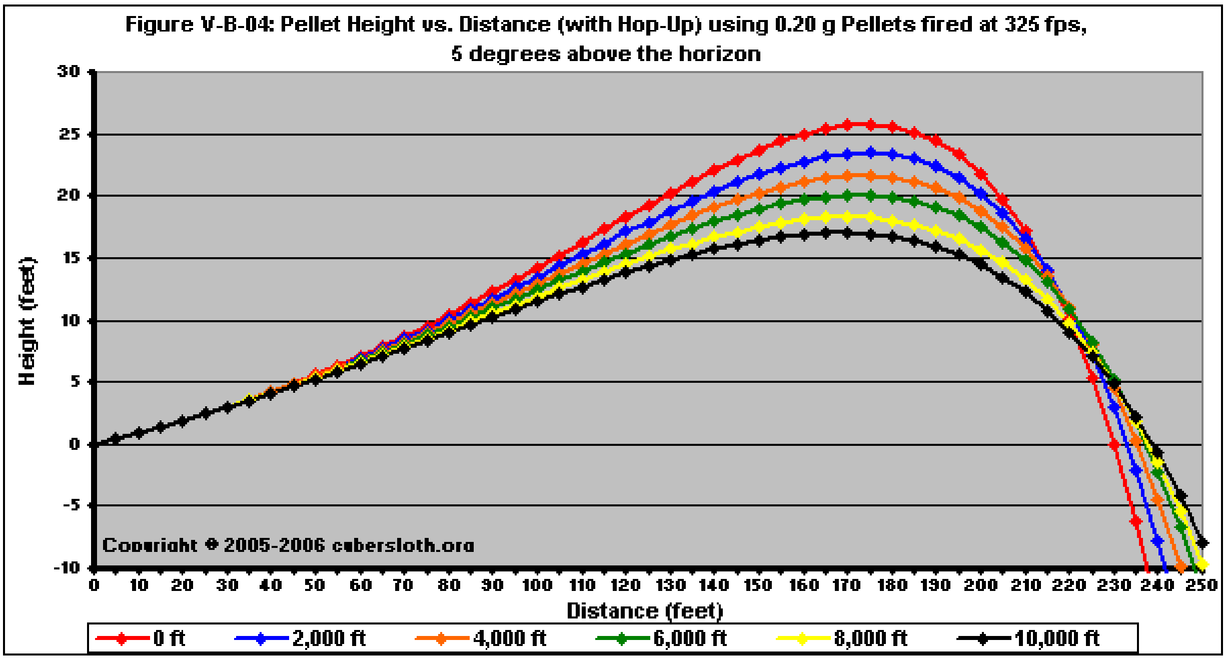
The effects are not that extreme. Going from sea level to, say, Denver, Colorado (5,300 ft msl), will only grant you an extra 5-10 feet or so of effective range, how much dependent upon muzzle velocity. At the same time, MED's would need to be increased 5-10 feet in Denver for high-powered rifles, somewhat offsetting the increased range. .

Figures V-B-02 and V-B-03 show how higher altitude shots maintain their velocity and energy better due to the reduced drag.





For shots fired level with the horizon, the lessened effects of hop-up at higher altitudes translates to a shorter trajectory. For shots fired at angles above the horizon, however, the diminished drag that the BB experiences allows it to have an ultimately longer trajectory, as depicted in Figure V-B-04.



The Airsoft Trajectory Project

Effect of Temperature on Trajectory

Section V-C: Effect of Temperature on Trajectory

Like altitude, temperature is a component of air density. In much the same manner that higher altitudes affect the trajectory of airsoft BB's, warmer temperatures will have a similar effect on the BB.

Cold air has a higher density than that of warm air. This affects the trajectory in two important ways. First, the BB is moving through a denser medium (compared to warmer air). Because of this, the BB will experience more drag along its trajectory in colder air, essentially reducing its trajectory. Additionally, while the denser air offers more mass for the momentum shift that would cause greater Magnus Lift, the denser air also causes the spin rate to decay more rapidly. The faster-than-normal spin decay ultimately limits the lift generated down range. This translates to a lessened trajectory for BB's fired in colder weather, regardless of whether they are fired with hop-up or without hop-up.

Figure V-C-01 shows BB height vs. distance for 0.20g BB's fired at 285 fps. As is clearly shown, warmer temperatures allow for longer trajectories. Ultimately, though, the difference is negligible. Even in the case given, the trajectories would be so similar out to 100' that the shooter would be unable to notice a difference.

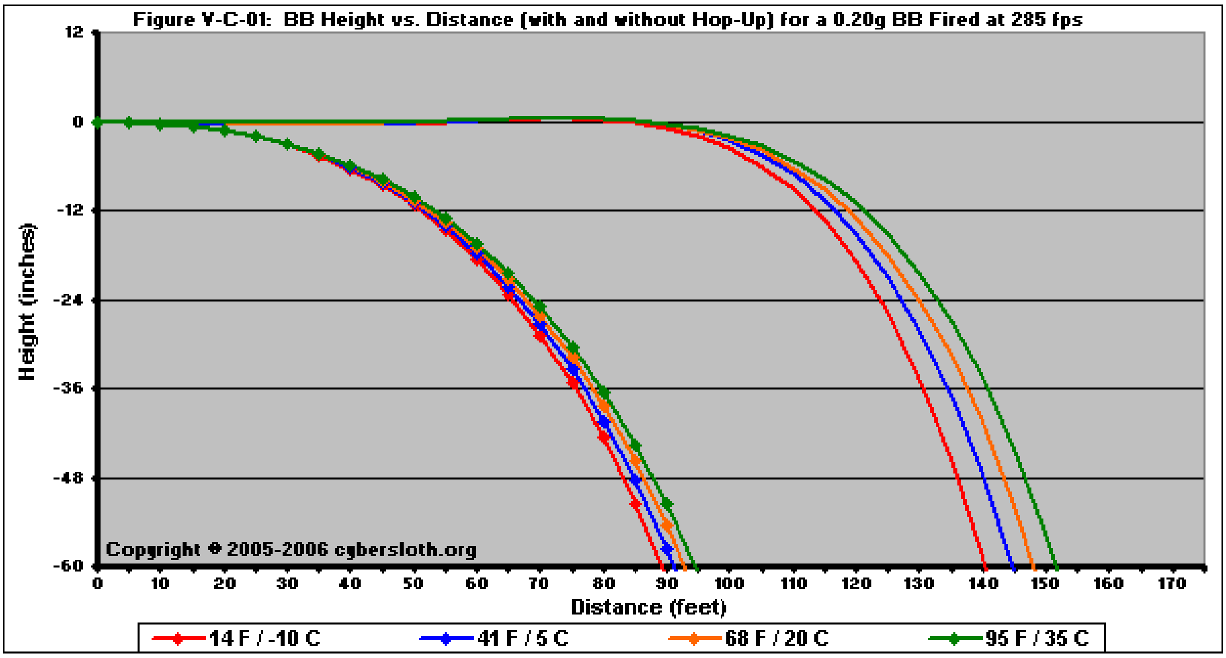
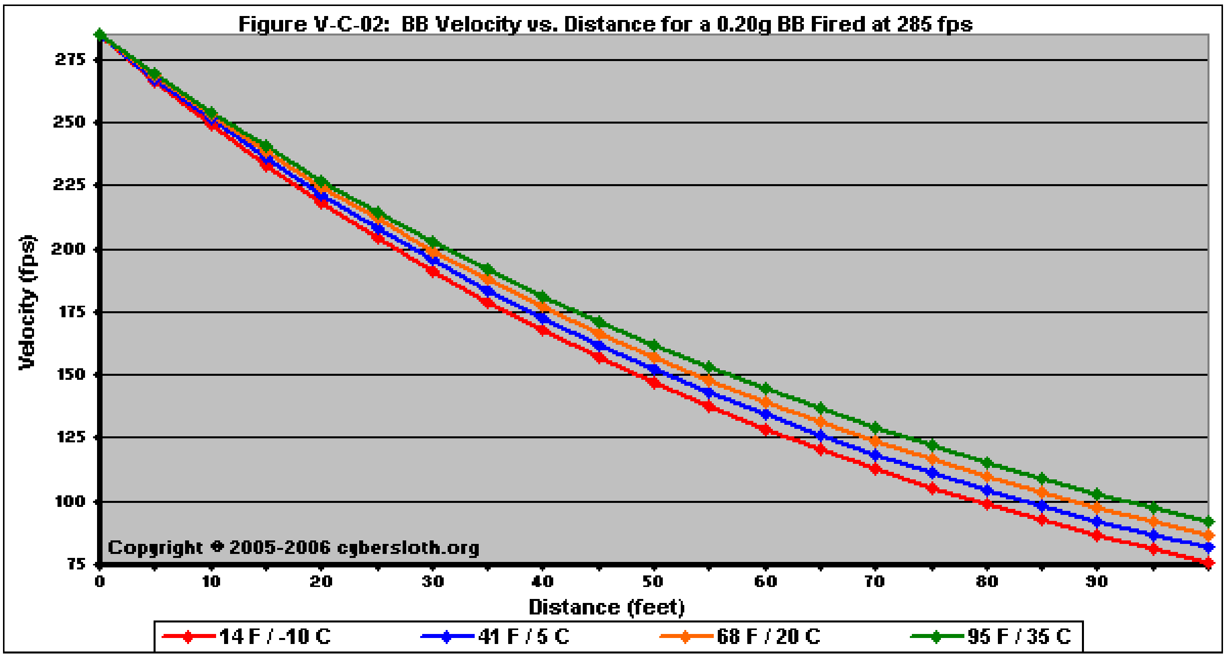
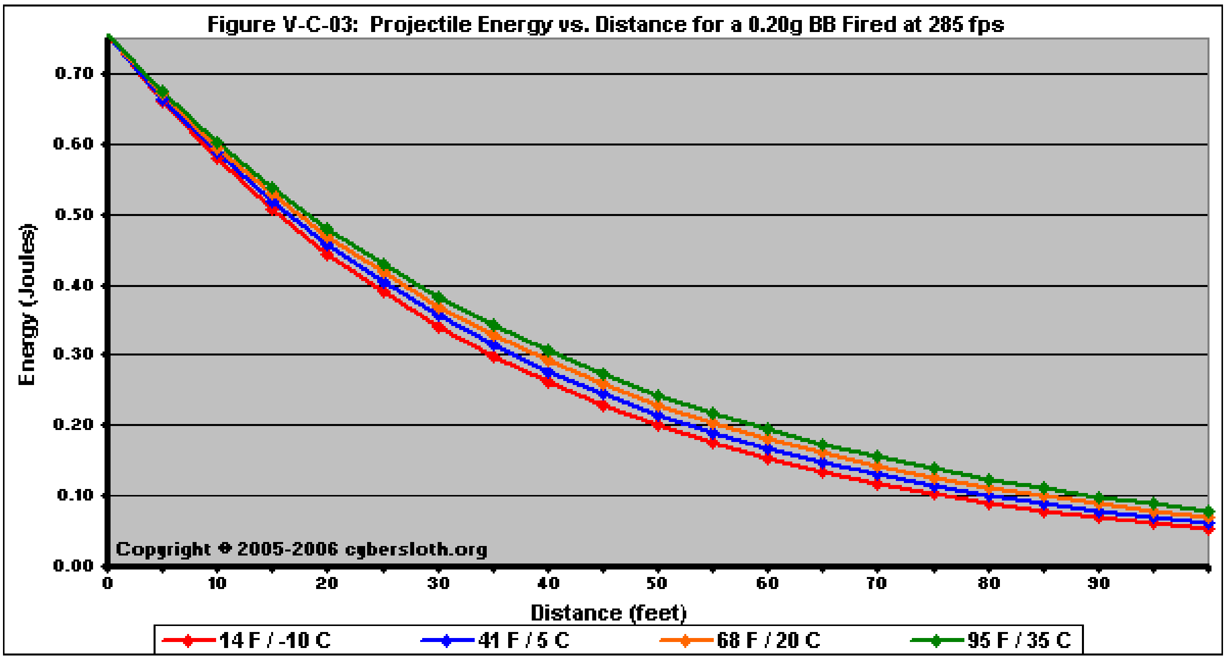


Figure V-C-02 shows the velocity decay versus distance. As expected, the BB's that are moving through the colder air experience more drag and ultimately have a lower velocity at a given distance. At a distance of 100 feet, a 0.20g BB fired at 285 fps will be moving at 110 fps in 95 degree weather. In colder weather, drag will have diminished the velocity more so. At 100 feet, a BB fired in 14 degrees will only be moving around 90 fps.



Diminished velocity translates into diminished impact energies. At a distance of 30 feet, a 0.20g BB fired at 285 fps will have an energy of 0.42 J in 95 F weather. By comparison, the same BB fired in 14 F weather will have an energy of only 0.38 J. Nevertheless, the disparity in energy between BB's fired in cold weather versus those fired in hot weather is negligible.



Keep in mind that these are theoretical explanations -- temperature has other effects on an airsoft rifle's performance. The difference in air density changes the compression in the cylinder during firing for AEG's and BAR's, while a colder ambient temperature diminishes the energy released from gas-powered rifles and pistols.

Ultimately, while altitude variations do translate into noticeable differences in trajectory and impact energies, the effects of temperature are relatively small. Keep in mind that it is only in extreme temperature differences that the effects are noticeable.

The Airsoft Trajectory Project

ATP - Minimum Engagement Distance

Section VI: Minimum Engagement Distance
Section VI-A: Determining Muzzle Energy

Provided that the mass of the BB is known and the muzzle velocity can be determined, muzzle energy can be easily calculated using the formulas as outlined in Section I-C: Energy. For most airsoft rifles and pistols, the muzzle energy for a 0.20g BB will be the same as for that of a heavier BB. As muzzle energies surpass 1.50 Joules, however, the muzzle energy correlation is not always equal. In high-energy guns, it is possible for the BB to accelerate and leave the barrel before the piston has had a chance to reach maximum compression. In this case, pressure will still be building as the BB exits the barrel; consequently the BB will not be able to fully absorb the energy from the piston. When a heavier mass BB is used in this gun, the BB will accelerate at a slower rate and remain in the barrel longer, thereby absorbing more energy.

An example to this would be a sniper rifle (TM PSG-1) recently tested:

BB Mass	0.20g	0.25g	0.30g	0.43g	0.88g
Muzzle Velocity (fps)	597.4	551.2	519.3	436.9	305.2
Muzzle Energy (J)	3.32	3.53	3.76	3.81	3.81

This is important to realize if guns are tested prior to events, as someone could arrive with a sniper rifle shooting near 600 fps with 0.20g and be given an MED based upon that muzzle energy, assuming that if the shooter was using 0.30 BB's, the rifle would be shooting at around 488 fps. As observed above, in reality the rifle might be shooting as much as 30 fps (and 0.40 J) higher than estimated. Consequently, a good recommendation is that a rifle be tested using the BB's that the shooter plans to use in the match as well as 0.20g.

Section VI-B: Safe Impact Energy

It is often stated that a 6mm projectile can pierce skin if the impact energy is greater than 1.35 J. Though I have not personally seen the document stating this, it sounds entirely plausible. Of course, there are other factors in play here, such as the angle at which the impact happens (straight-on vs. a glancing shot), as well as the place on the body where the impact occurs (as skin thickness and strength varies over one's body). What doesn't sound right is when I read someone stating that it is actually an energy of something (such as 4.00 Joules) and that this has been determined by "leading experts" and applies to all objects. It is the caveat that the listed energy "applies to objects of any diameter" that is clearly an errant claim. People forget to take into account the object's size when they are trying to determine what constitutes a safe impact energy. A projectile's size is important because it determines over how much area the impact will be distributed.

Simply stated, objects with a smaller diameter will be able to pierce skin at a given impact energy more easily than that of a a larger-diameter projectile. An easy way to think about it is to take two objects of different diameter (size) and consider the impact by each at equal energies.

A 0.20g, 6mm BB moving at 656 fps (or 4.00 Joules) will easily pierce skin. By comparison, a regulation golf ball (1.62 ounces, or 46g) moving at 75 fps (which is around 50 mph) will have an impact energy of about 12 Joules. However, a golf ball moving at 50 mph will not leave a bruise, much less pierce skin.

The reason for which the 6mm BB will pierce skin yet the 42mm golf ball will not is that the golf ball's impact energy is distributed over a much larger surface area. For the 6mm BB, the energy is distributed over a frontal area of about one-twentieth of a square inch, whereas the golf ball's energy is distributed over a frontal area of over two square inches. So when we discuss impact energies, it is important to realize that impact energy alone will not tell us whether or not the impact is safe, we must consider both impact energy and impact area.

One could argue that the golf ball would not penetrate due to the lower speed, however, again it is important to stress that it is the *impact energy* relative to the *impact area* that causes penetration; and not just impact energy alone. Since all 6mm BB's have nearly the same orthogonal area (i.e., impact area), we can focus on impact energy by itself since area is taken as a constant.

While that may seem like superfluous information, it is important to recognize that due to the extra surface area and consequent distribution of impact energy, 8mm BB's are inherently safer than 6mm BB's. For designated snipers firing 6mm projectiles, I prefer a minimum engagement distance based upon an impact energy of 1.00 Joule. Because a 8mm projectile distributes the impact energy of a significantly greater area (as the cross-sectional area nearly twice as large), a higher impact energy could probably be used for determining MED's, however without any supporting data, I felt that it safer recommendation was to use the 1.00 Joule limit for 8mm BB's as well.

To get below 1.00 Joules, a 0.20g BB would need to moving at less than 328 fps. Each BB weight class has a specific maximum impact velocity:

BB Weight Class (g)	Max Impact Velocity (fps)	Max Impact Velocity (m/s)
0.20	328	100
0.25	293	89
0.30	268	82
0.36	244	75
0.43	224	68
0.34	252	77
0.45	219	67

Keep in mind that a safe impact energy is independent of size and only dependent upon mass and velocity; a 0.34g 6mm BB will have the same safe maximum impact velocity as a 0.34g 8mm BB (in this case, 252 fps), if the maximum impact velocity of each is to be taken as 1.00 Joule..

The Airsoft Trajectory Project

ATP - Closing Remarks

Section VIII: Closing Remarks

The study of the BB trajectories has been both challenging and interesting. There were a lot of questions that I didn't know the answer to when I set out to create the Airsoft Trajectory Project, and still other questions that I thought I knew the answer to, only to find out that I was wrong. It has taken a lot of testing, a lot of help, and a lot of communication to come up with the right model parameters. All in all, I'm fairly happy with the results as well as the learning process, and hope that others will find it useful too.

If you've waded through the data and haven't been able to figure out the answer to the initial questions posted on the introduction page, here they are again, complete with short (or as short as possible) answers and links to more in-depth explanations.

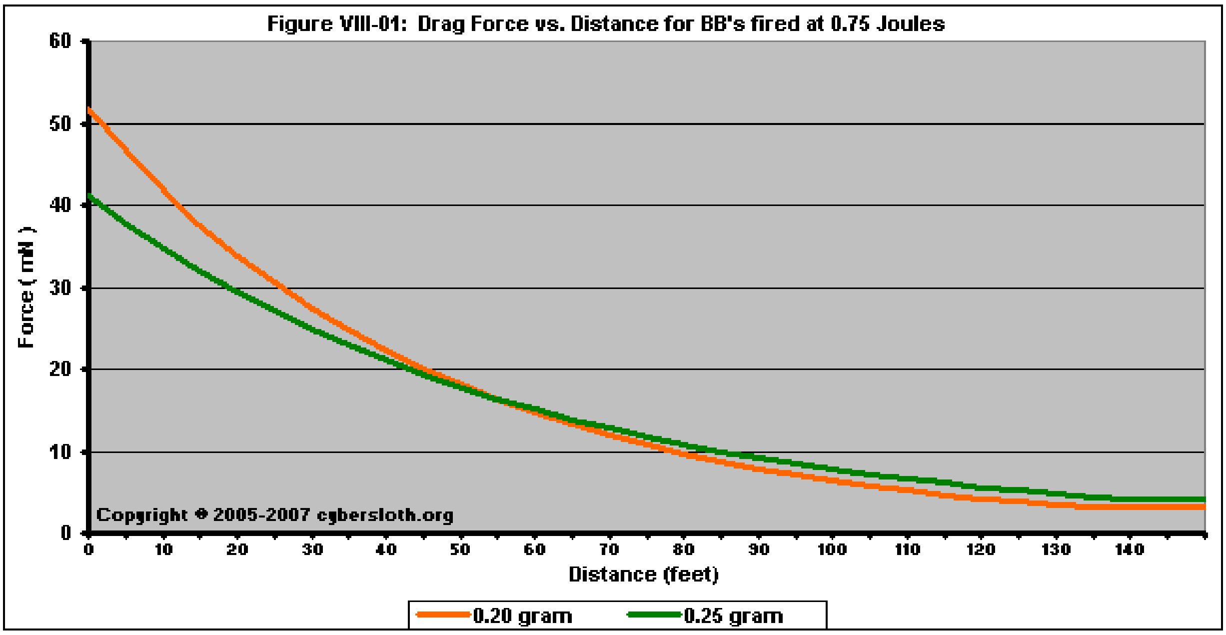
Is it worth upgrading a gun from x fps to y fps to get more range?

There isn't a simple answer to this question... it ultimately depends on individual preferences. Hopefully all of the data will help you to determine whether or not it is worth upgrading.

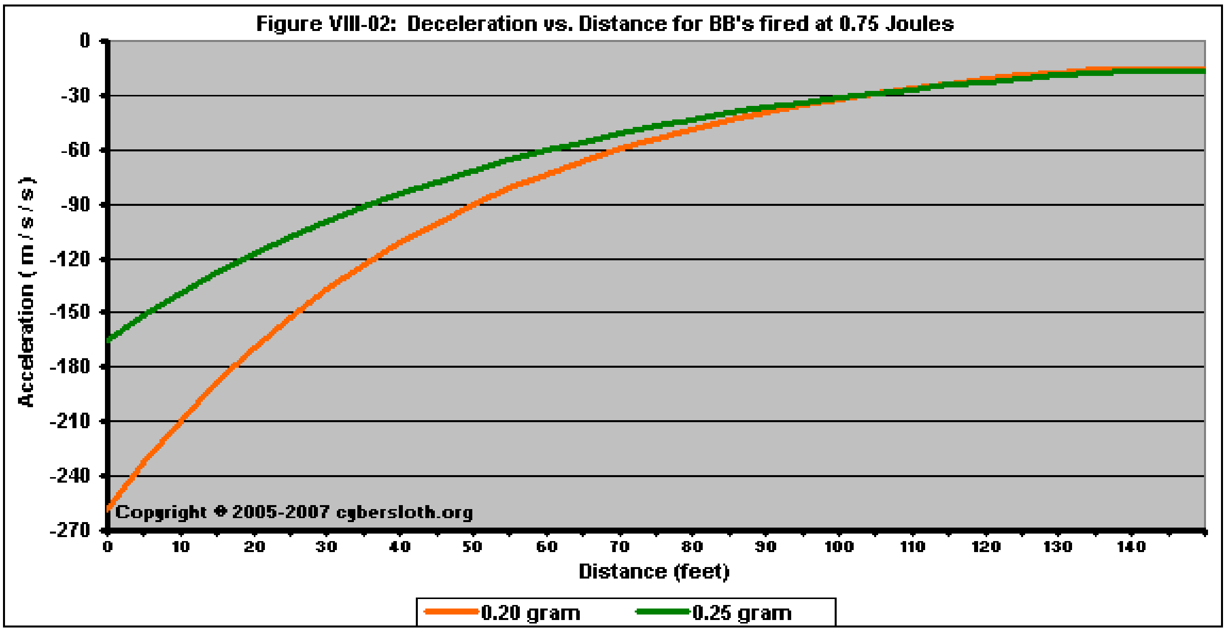
For equal muzzle energies, which BB goes further, 0.20g or 0.25g?

One of the classic, seemingly never-ending debates that comes up is that of whether or not 0.20g BB's outrange 0.25g BB's. Given equal muzzle energies for BB's fired without hop-up, the heavier BB will always outrange the lighter one. There are two reasons for this: first, for a given equal muzzle energy, a lighter BB will have to be moving faster than the heavier one. Since the force of drag goes up exponentially with respect to velocity, a higher velocity will provide a much higher drag force than a lighter one. Which is to say that, for equal kinetic energies, the force of drag experienced by a lighter projectile will be much higher than it will be for a heavier projectile. Additionally, mass is an inverse component for calculating the deceleration due to drag. In other words as the mass of the projectile is increased, the deceleration due to drag is decreased thus extending range.

To illustrate this, here are charts depicting the deceleration and drag force experienced by a 0.20g and a 0.25g BB fired at 0.75 Joules (or 285 and 255 fps respectively). In Figure VIII-01, we can see that the drag experienced by the 0.20g BB is initially much greater than the drag experienced by the 0.25g BB. At about 53 feet, the drag force for both BB's are equal. This is simply because the drag equation does not take into account mass, but instead is primarily governed by velocity. At a distance of about 53 feet, both BB's are moving at about 165 fps, consequently they are experiencing the same drag force at that distance.



Acceleration, however, is dependent upon mass. Out until about 100 feet, the 0.20g BB experiences a greater deceleration than the 0.25g BB, and initially, the 0.20g BB experiences a deceleration that is 60% greater than that experienced by the 0.25g BB..

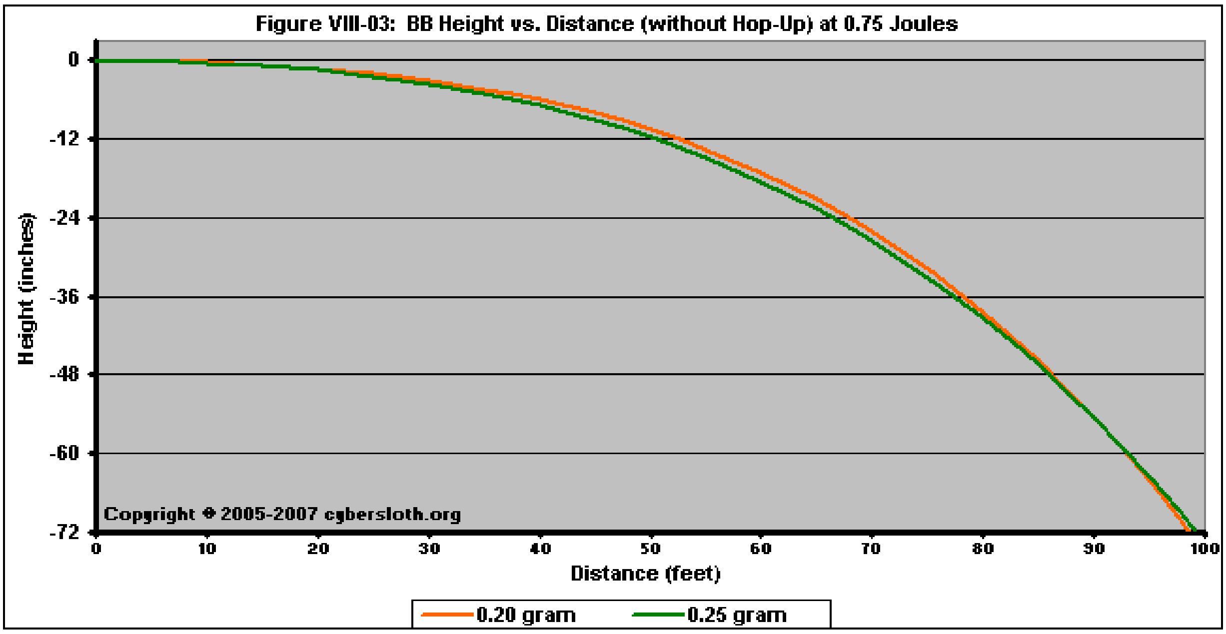


All of this may seem a little confusing because we're discussing both *force* and *acceleration*. To put it in perspective numerically, here are the calculations for instantaneous drag and instantaneous acceleration for three different BB's, Projectiles A, B, & C:

Projectile	Mass	Velocity	Energy	Drag Force	Acceleration
A	0.20 g	285 fps	0.75 J	52 mN	-258 $^{m}/_{s}2$
B	0.25 g	255	0.75	40	-162
C	0.25 g	285	0.94	52	-206

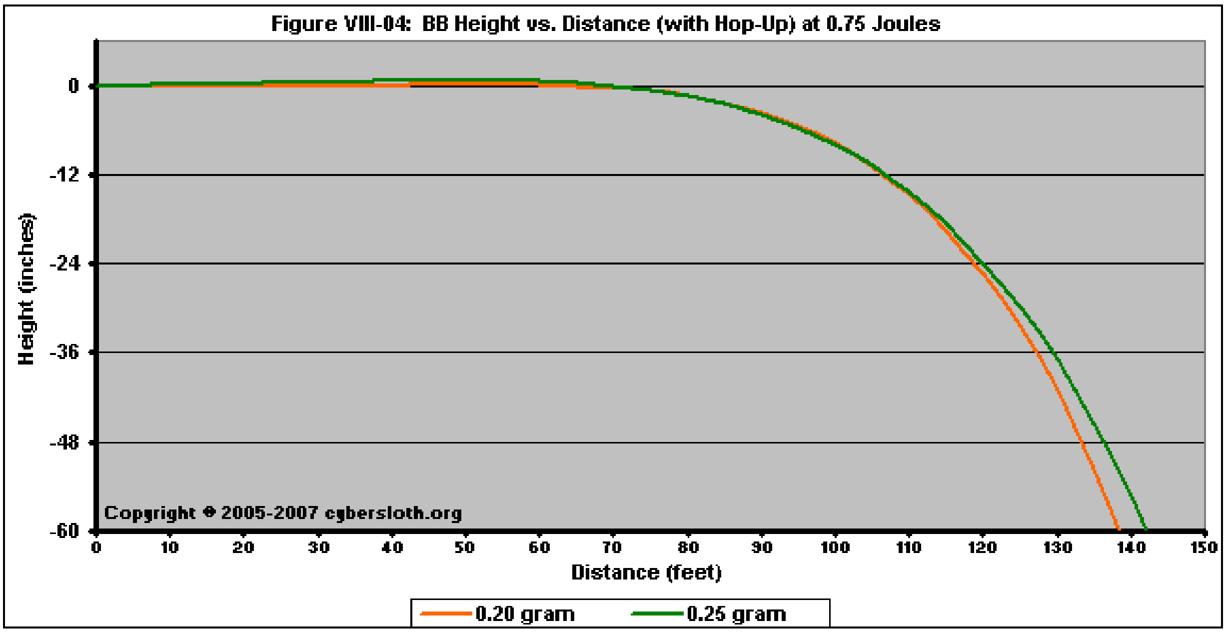
Projectiles A & B are fired with equal energies. Because B has greater mass 0.25g, it achieves 0.75 Joules at a slower velocity compared to the 0.20g mass of A. Because B is moving slower, it experiences less drag force (which in turn contributes to a lesser deceleration). Note, however, that Projectiles A & C are moving at the same velocity (and because C has more mass, it has more energy). The drag forces, which are independent of mass, are equal for both BB's at the same velocity. The deceleration of C, however, is less because acceleration *is* a function of mass.

In Figure VIII-03, we can see that for non-spinning BB's, the 0.25g BB's eventually outdistance their lighter counterparts.. However, the thing that is most important to observe is that the difference is so small that it would be nearly impossible for the naked eye to observe.



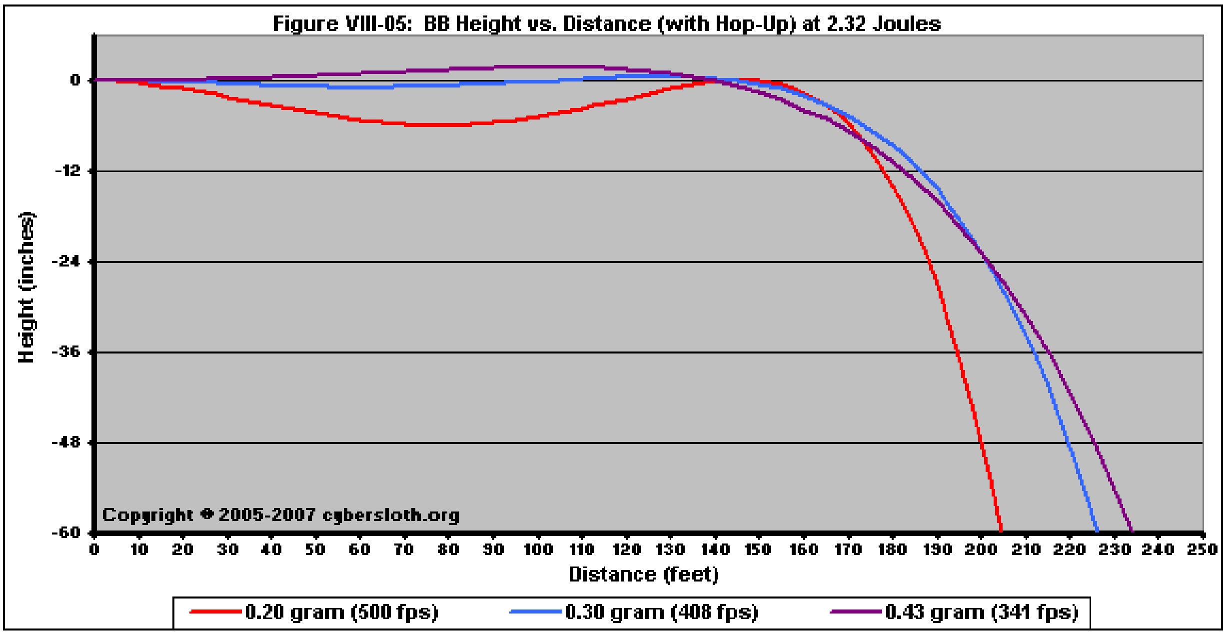
When hop-up is applied, the lighter BB's are able to use backspin to generate lift more so than the heavier BB's. Of course, lighter BB's also lose spin faster than heavier ones, meaning that the effects of lift drop off rapidly down range. Ultimately, what happens is that both types of BB's experience similar trajectories.

In other words, there isn't an easy answer to the question of which BB goes further. Frankly, a lot of the answer comes down to semantics, namely what we describe as "effective range." In Figure VIII-04, we see the effects of hop-up on the same 0.20g and 0.25g BB's as above. If we consider "effective range" as the distance at which a BB experiences the least amount of deviation from the aim point out to 100 feet, then both BB's perform similarly. If a person is using a sniper rifle and is aiming at a target that is 140 feet away, and considers this the effective range for their sniper rifle by using holdover, then the 0.25g BB will travel further. Again, there isn't a clear answer, but rather depends on what the shooter defines as "effective range." (And... it would be hard to see a difference in trajectory between the two BB's as the trajectories are very similar.)

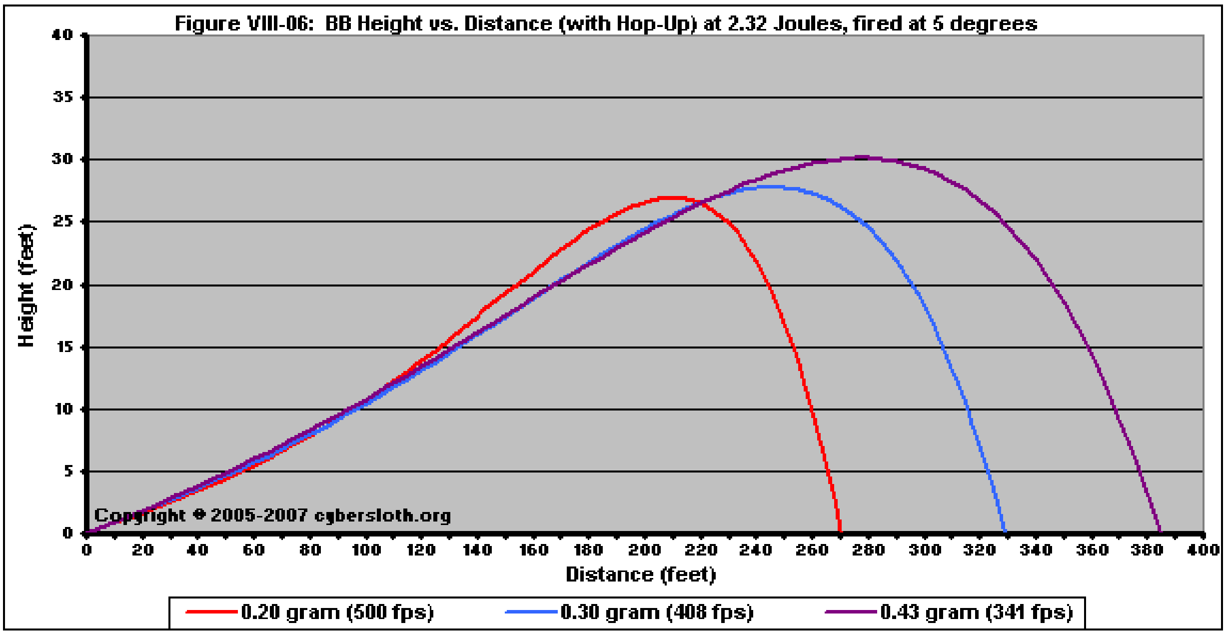


What is most important is that heavier BB's do not translate to a loss in range. At worst, the trajectory is on par with their lighter counterparts, and more often than not, heavier BB's will outrange lighter ones.

In Figure VIII-05, we see the trajectories of 0.20g, 0.30g, and 0.43g BB's fired at 2.32 Joules. At such a higher power, 0.20g BB's experience wild trajectories. If the shooter is aiming above their target at over 225 feet, 0.43g BB's do indeed go further, but it is the 0.30g BB that produce the desired trajectory.



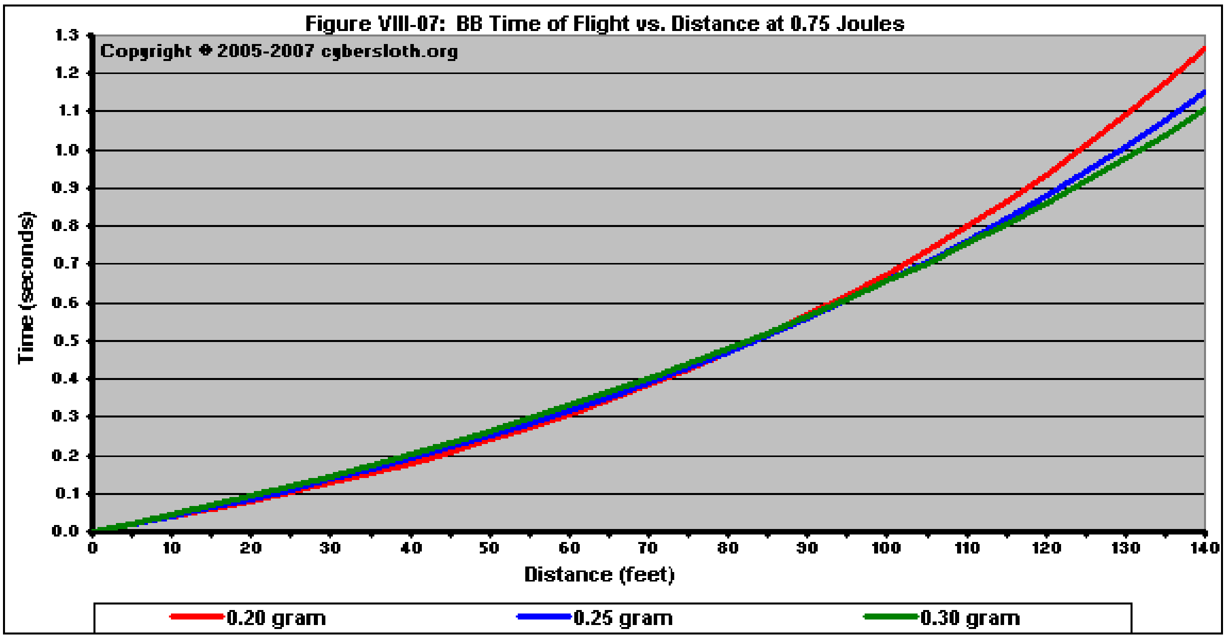
A better example of the way in which heavier BB's outdistance lighter ones is to look at trajectories of a various masses fired at an angle above the horizon. Although of no practical importance, in Figure VIII-06 we can clearly see that heavier BB's do produce greater ranges.



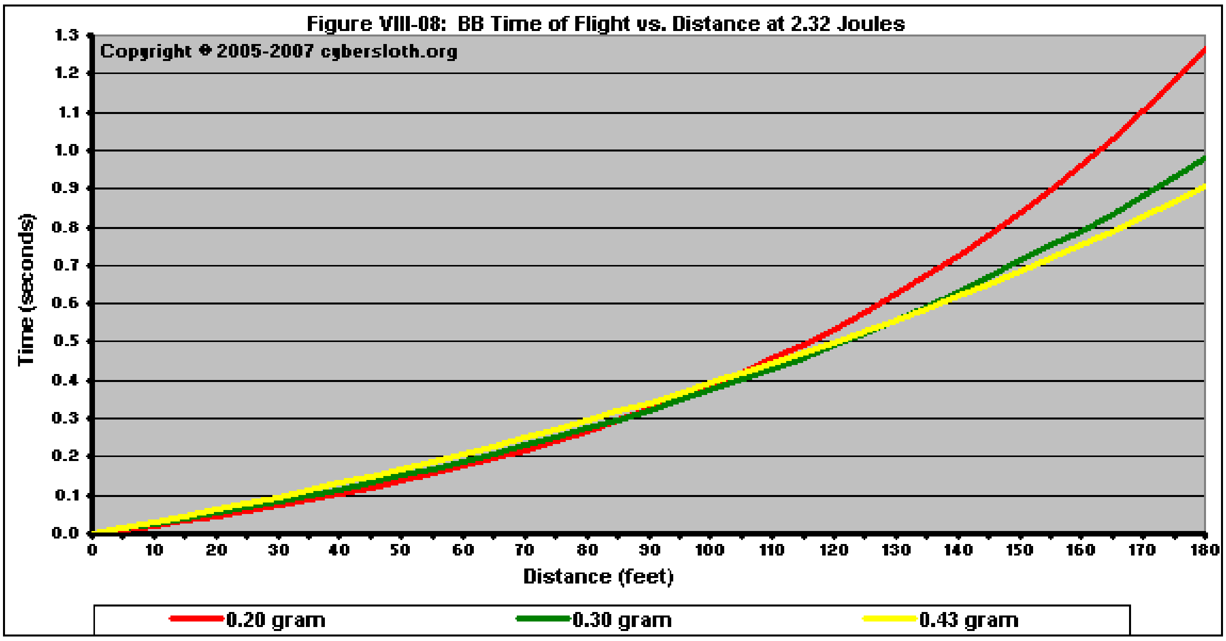
Ultimately, heavier BB's will outrange their lighter counterparts. This does not mean that heavier BB's are always the most useful or necessary ammunition.

<u>Which mass BB gets to the target the quickest for the given velocity?</u>

At ranges less than 100 feet, all BB's weighing between 0.20g and 0.43g get to their targets at roughly at the same time (assuming equal muzzle energies). In Figure VIII-07, we can see the flight times for various BB's fired at 0.75 Joules. All of them reach their targets at roughly the same time. For instance, a 0.20g BB reaches 70 feet in about 0.37 seconds, while a 0.25g BB reaches the same target in about 0.38 seconds, the time difference being hardly worth noting. At around 110 feet, we can see that the heavier projectiles are reaching their targets slightly faster. At 140 feet, a 0.25g BB would reach its target about 0.10 seconds faster than a 0.20g BB. Bit again, the difference is negligible.



In high-powered rifles, the difference in time of flight is hardly noticeable at ranges less than 120 feet. In Figure VIII-08, we can see that the time difference only becomes really noticeable at 155 feet, where a 0.30g BB would reach its target roughly 0.20 seconds faster than a 0.20g BB. Again, this is noticeable on the chart, but 0.20 seconds goes by rather fast when being shot at.



So if you wanted a short and easy answer, it would be this

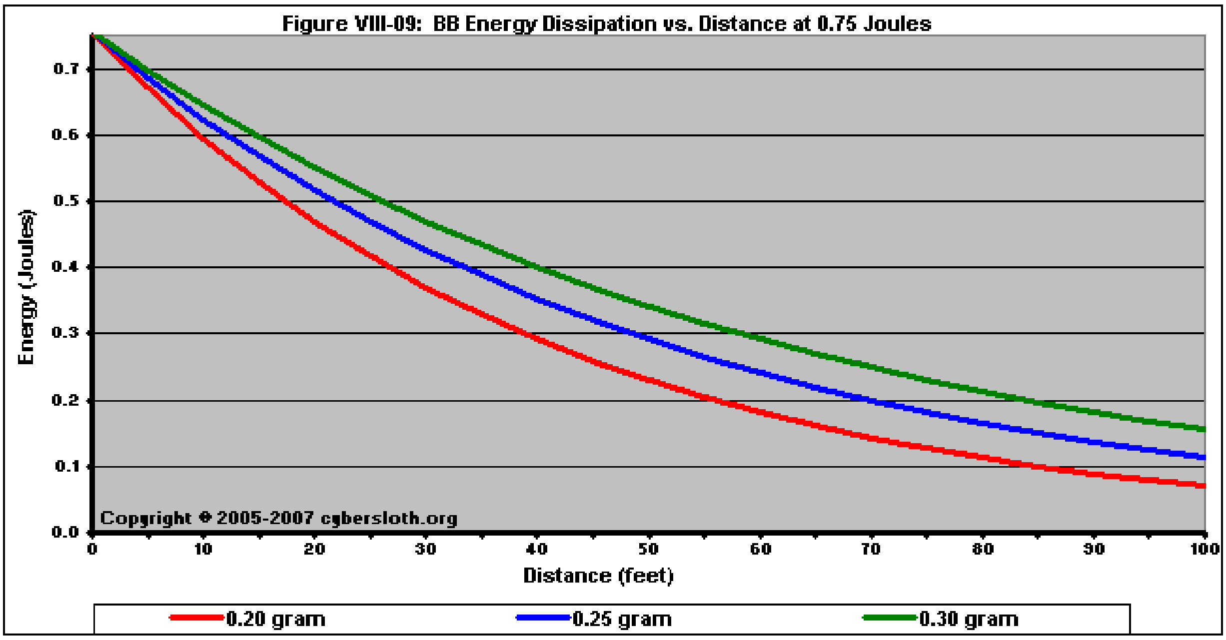
At ranges less than 125 feet, all BB masses reach their target at the same time if fired with equal muzzle energies. Beyond 125 feet, the heavier BB will usually get to the target faster than the lighter one.

Do heavier mass BB's have more energy than lighter BB's down range?

Determining which BB mass reaches its target the quickest or which BB mass goes furthest requires calculations and a complicated answer. Determining whether heavier BB's retain their energy better down range, however, has an easy and straight-forward answer that will always be true:

Assuming that a heavy and a light BB are being shot with equal muzzle energies, the lighter BB will always, always, always experience faster energy dissipation.

As was explained above, for a given muzzle energy a lighter BB will begin traveling faster. Because of this, it will decelerate much more rapidly compared to a heavier BB. We can see the energy dissipation in Figure VIII-09. In this case, we're looking at a various BB masses fired from a stock Tokyo Marui rifle with a muzzle energy of 0.75 Joules (with the 0.20g fired at 285 fps, the 0.25g fired at 255 fps, and the 0.30g fired at 233 fps). Notice that the heavier BB's retain their energy much better.



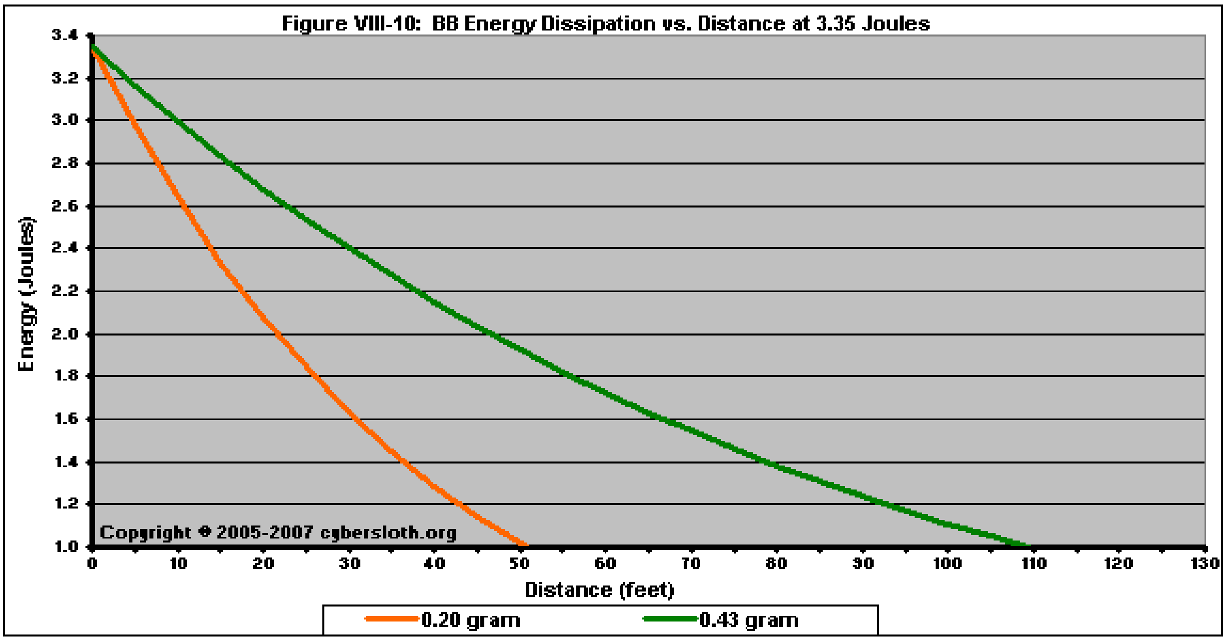
This is very important when determining safe engagement distances, and is explained in greater detail in Section VI: Minimum Engagement Distance.

Keep in mind that this only holds true when comparing 6mm BB's to other 6mm BB's. The area normal changes when talking about 8mm BB's, however a 0.34g 8mm BB will always lose energy much more rapidly compared to a 0.45g 8mm BB.

Is it necessary to restrict a rifle with a 600 fps muzzle velocity to a minimum engagement distance of 100 feet?

As was discussed above (and discussed very thoroughly in Section VI-C: Recommended Universal MED's), ***it all depends on what BB the shooter is using.***

Let's take the example of a rifle firing at 3.35 Joules, with energy dissipation depicted in Figure VIII-10. If a site wishes to base their MED's using a maximum impact energy of 1.00 Joules, then a shooter firing a 0.20g BB at 3.35 Joules (or 600 fps) would need an MED of only... 50 feet. However, if the same shooter were firing a 0.43g BB at 3.35 Joules (or 409 fps w/ 0.43g), then the MED would need to be extended out to... 108 feet, over twice the distance needed for the 0.20g shot.



The main reason that I divide MED's based on BB weight is that weight (or mass, technically, but you know what I mean) makes a huge difference in energy dissipation. If a site wishes to enforce an MED for rifles that chrono at 500 fps with 0.20g BB's, then they could either restrict the MED to 80 feet for any BB weight, or use an MED of 55 feet and restrict the maximum weight of the BB's to 0.30g (though all snipers would be on the "honor system" in terms of ensuring that they do not use a BB weight above 0.30g).

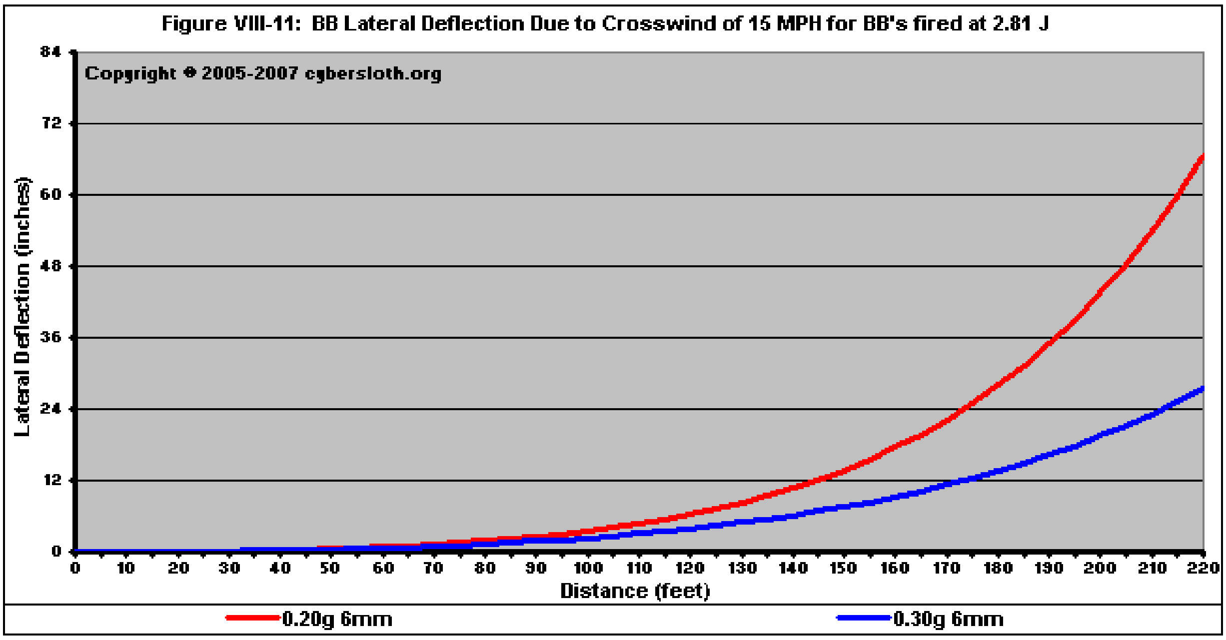
To see how different BB weight classes translate to different MED's, use the calculator provided in Section X-B: Relative Energy / MED Calculator. Even a 1.49 J rifle (400 fps w/ 0.20g) has different MED's whether using 0.20g or 0.25g BB's.

What MED's are recommended to ensure both safety and fairness to all shooters?

Again, it is a complicated answer, however it is answered in full in Section VI-C: Recommended Universal MED's, complete with printable charts. If you have any questions, or would like advice or custom charts for your site, feel free to contact me.

Do 0.43 gram BB's negate the effects of wind *that* much better than 0.20 gram BB's?

This is another easy one. Heavier BB's always hold their trajectory better than lighter ones in the wind. They do so for the reasons discussed above wherein we learned that acceleration is inversely proportional to mass. As the mass of the projectile increases, the lateral acceleration decreases meaning that heavier BB's will experience less drift compared to lighter ones. This is explained in detail in Section V-A-02: Crosswind Component and is further depicted below in FigureVIII-11.



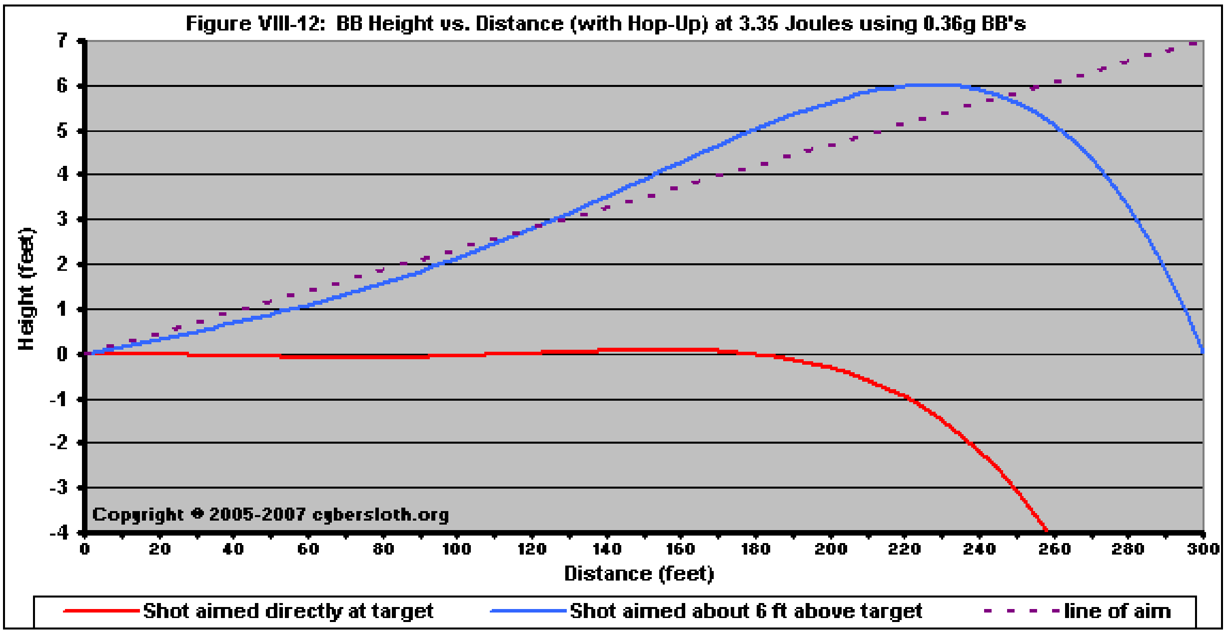
Here we can see the effects of a 15 mile per hour crosswind on a 2.81 Joule rifle. At a range of 220 feet, a 0.30g BB will have drifted a little over two feet lateral to the target, whereas a 0.20g BB will have been deflected over five feet lateral to the target. Going to progressively heavier-mass BB's would produce even less lateral deflection, however it would observe a law of diminishing returns, as is shown in <u>Section V-A-02: Crosswind Component</u>.

<u>Are people really able to achieve ranges out to 300 feet?</u>

In short, it is definitely possible to hit something at 300 feet.

Having worked through the problem in terms of theory and some testing using my own rifle, it is very possible to hit something at 300 feet and airsoft snipers are indeed achieving "kills" at that range and beyond. That being said, I'm still pessimistic in that I doubt that a shooter could *routinely* hit a target at that range. The longest range that I have achieved, observed, or have had verified for hitting at a 18" diameter target consistently is 235 feet. That is not to say that further engagements are impossible, but rather that this is the highest effective range that I have observed first-hand. Your mileage may vary...

If a shooter has a highly-upgraded rifle, he can extend his range out to 300 feet by aiming above the target. Take for example a shooter with a 3.35 J rifle firing at a target 300 feet away. Figure VIII-12 shows the trajectory of a 3.35 J shot using 0.36g BB's. In this case, the shooter would need to aim nearly seven feet above their target. Again, the rifle would have to be very consistent, and the shooter would probably need luck on their side, ***but it is definitely possible***. (And... a 300' shot would be highly, *highly* unlikely with either a 0.20g or 0.25g BB.)

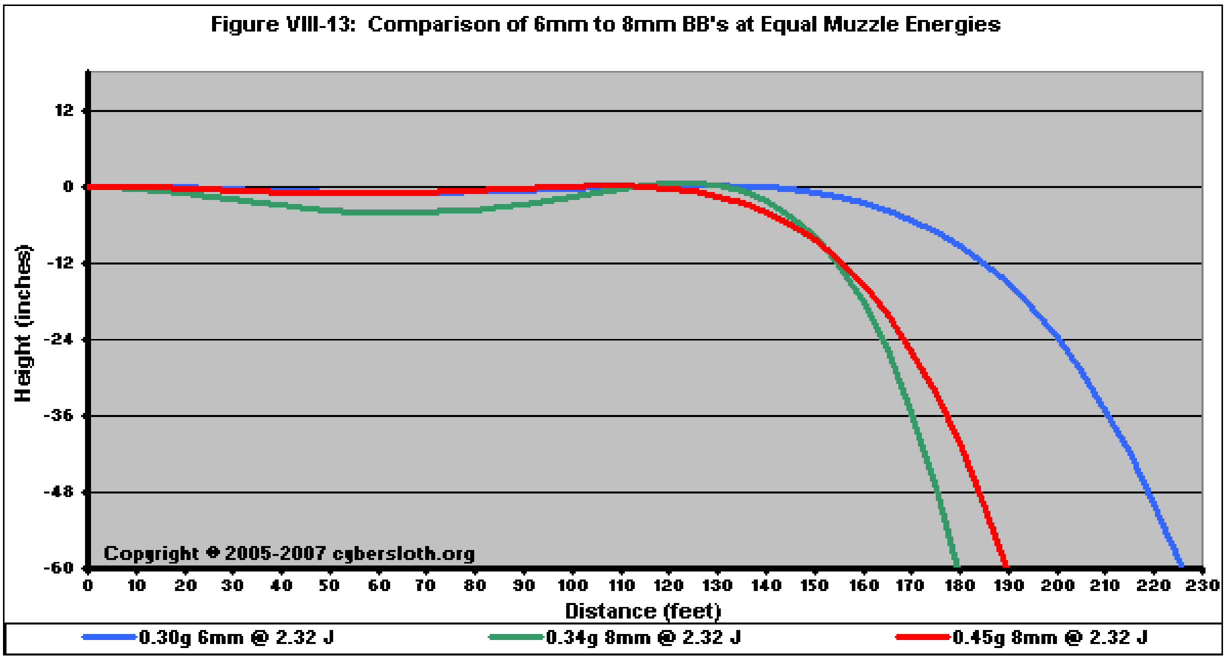


Another thing that a shooter should realize is that the impact energy at 300' will be greatly, greatly reduced. In the case of the 3.35 J rifle firing a 0.36g BB, the impact energy of that shot at a range of 300' would be about 0.06 Joules. In our tests, we found that BB's hitting a target at 0.06 Joules bounced off of standard sheet of paper (typically without leaving even a mark) and would be all but impossible to feel.

Do 8mm BB's provide better range than 6mm BB's?

The short, simple answer is "nope."

However, like most things, this is a subject that requires a little interpretation. First up is a comparison of BB's fired at 2.32 Joules, the equivalent of 500 fps with 0.20g BB's. For the examined BB's, this would be equivalent to 408 fps with the 0.30g 6mm, 384 fps with 0.34g 8mm, and 333 fps with 0.45g 8mm. Figure VIII-13 depicts the trajectories:



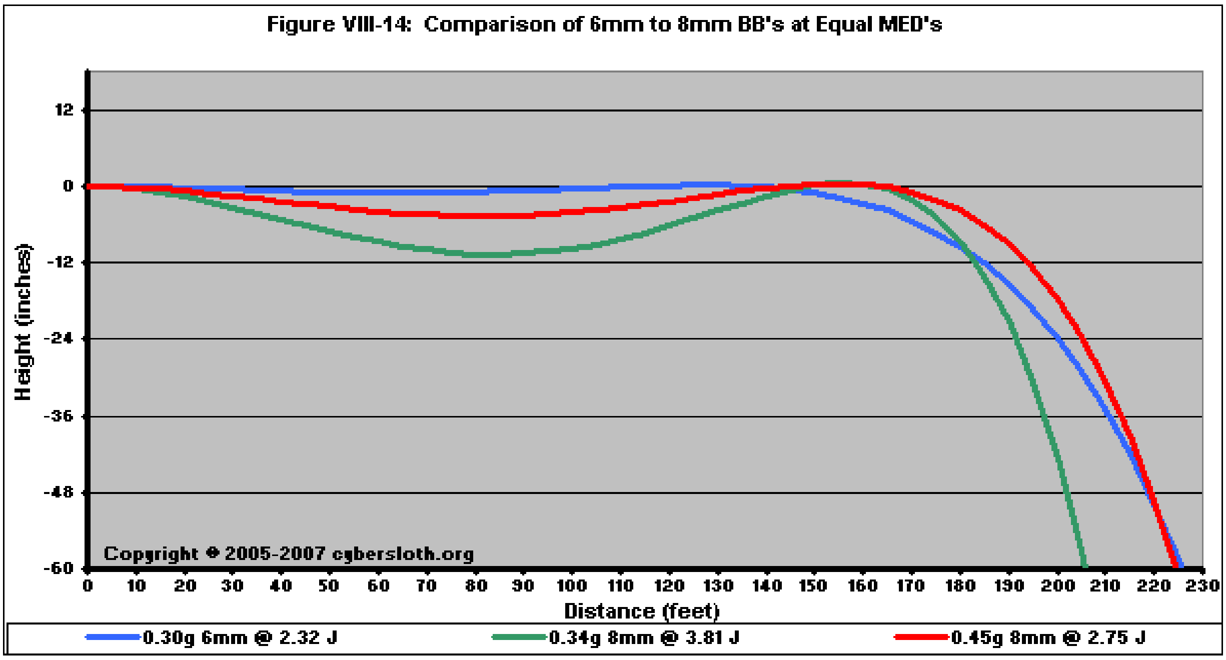
It is fairly obvious that, for equivalent muzzle energies, the lighter 6mm BB actually produces greater range. More specifically, the 6mm BB goes about 25 feet further than the 0.45g 8mm BB, and about 30 feet further than the 0.34g 8mm BB.

One thing that needs to be considered here is the fact that 8mm BB's dissipate velocity -- and consequently energy -- at a much faster rate than 6mm BB's. In other words, the 6mm BB's in the above case require greater MED's. The 0.30g 6mm BB would need an MED of 53 feet, whereas the 0.45g 8mm BB would need an MED of 45 feet, and the 0.34g 8mm BB would need an MED of only 38 feet.

One problem that I see throughout U.S. sites is that 8mm BB's are given equivalent, or in some cases greater, MED's when compared to 6mm BB's. Imagine a site that restricts muzzle velocities to lower than 500 fps, and gives an MED for those rifles of 55 feet. An airsoft player shows up with an 8mm rifle that fires a 0.34g 8mm BB at 487 fps (a whopping 3.75 Joules). Most sites would not allow this rifle on the site. In reality, if the site considers a 500 fps rifle fine with an MED of 55 feet, then the 487 fps would be just as safe with the same MED; at 55 feet, both would have an impact energy of 1.00 Joule (and the 8mm BB would ultimately be safer still because the energy is dispersed over a greater area). (To learn more about MED's, and to see recommended MED's based on muzzle energy, BB weight and size, consult Section VI-C: Recommended Universal MED's).

All of that is to say that comparing 6mm BB's to 8mm BB's based on muzzle energies puts the 8mm BB's at an unfair disadvantage. (And keep in mind that I am not an 8mm shooter, and do not make these statements based on bias but rather on an effort to ensure both safety and fairness in airsoft.) A better comparison of 6mm BB's to 8mm BB's would be one based off of the MED's imposed.

As previously stated, a 500 fps rifle firing 0.30g 6mm BB's would have an MED of 55 feet. For a 0.34g 8mm BB to have an MED of 55 feet, it would need to be fired at 487 fps (3.75 J), and a 0.45g 8mm BB would need to be fired at 360 fps (2.71 J). Figure VIII-14 depicts the trajectories of the three BB's.

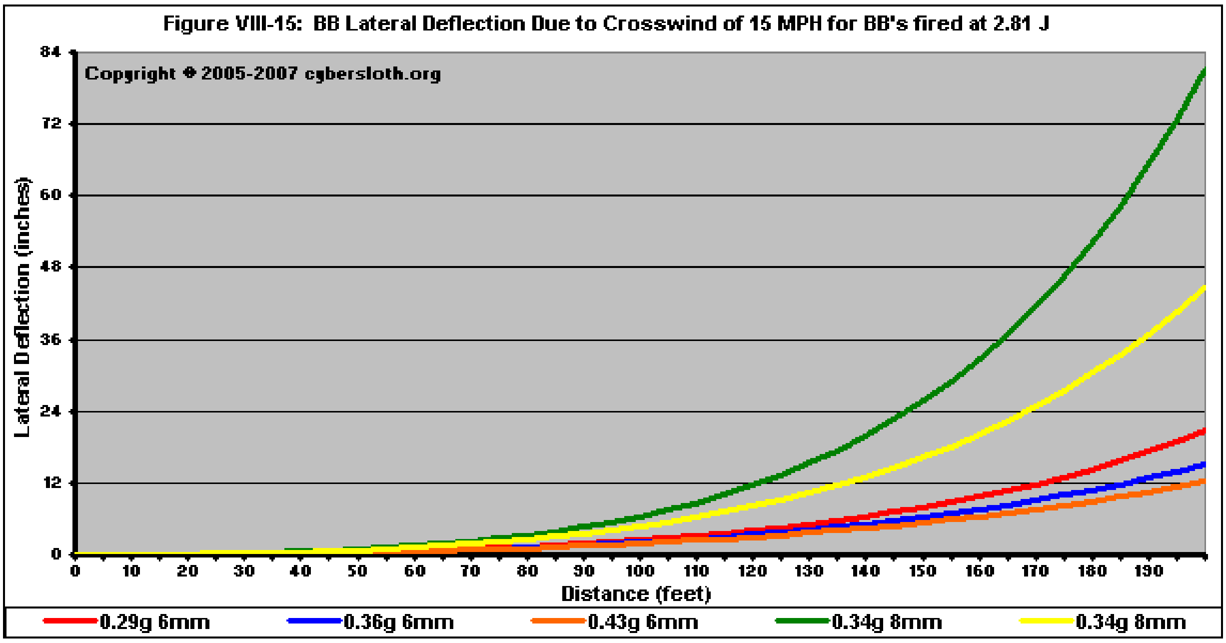


Even with equal MED's, the 6mm BB still outperforms the 8mm BB's, however both are a lot closer now. Because 8mm BB's are more consistent and require lower spin due to the hop-up to achieve the desired trajectory (and because 8mm BB's are less likely to fly errant due to surface imperfections), the 8mm BB's would achieve greater consistency in the terminal phase of their trajectories. Even so, given the added range (and precision standard of some high-grade 6mm BB's), I would consider the effective range of 6mm BB's to always be greater than that for 8mm BB's.

Do 8mm BB's resist wind better than 6mm BB's?

For the same reason that 8mm BB's dissipate velocity at a much greater rate than 6mm BB's, 8mm BB's also experience greater lateral deflection due to a crosswind. One thing to keep in mind is that 8mm BB's are less dense in comparison to 6mm BB's (for specifics, see Section I-A-02: Density/Volume). Additionally, the greater sail area of the 8mm BB's means that the lateral drag due to wind is going to be higher than that for 6mm BB's.

Figure VIII-15 depicts the lateral deflection of various 6mm and 8mm BB's fired at 2.81 J in a 15 mile-per-hour cross wind.



At a range of 200 feet, the 6mm BB's have only been deflected between one and two feet from the aimpoint. The 0.45g 8mm BB, by comparison, has been deflected nearly four feet, and the 0.34g 8mm BB has been deflected nearly seven feet. Even if you were to compare the 0.45g 8mm BB to a 0.20g 6mm BB, the 0.20g 6mm still holds it's trajectory better in terms of lateral deflection (the 0.20g 6mm BB deflection is above).

In brief, **6mm BB's resist the effects of crosswind MUCH better than 8mm BB's.**

Do high-velocity BB's resist wind better than low-velocity BB's?

High-velocity BB's DO NOT resist wind any better than low-velocity BB's. Rather, high-velocity BB's reach their target faster than low-velocity BB's, meaning that there is less time for the wind to deflect a high-velocity BB.

To better explain it, here is a graph with text taken directly from Section V-A-02: Crosswind Component:

Figure VIII-16 shows how the relationship between muzzle velocity and crosswind. We can see that for equal BB weights, a high-power shot will reach its target having experienced less deflection. For instance, if you were firing at a target 100 feet away with a 20 mph crosswind, a 0.20g BB fired at 400 fps will have only been deflected 11 inches, whereas a 300 fps shot would have been deflected around 19 inches.

Keep in mind that for an equal time period, the BB moves the same distance laterally for each shot. The high-power shots do not resist crosswind any better than the low-power shots; rather, the high-power shots get further down range in less time. To explain it further, the 600 fps shot reaches 100 feet after 0.32 seconds, after which time it has been deflected about 5 inches. After 0.32 seconds, the 500 fps shot has only gone 90 feet, where it has an equal deflection of 5 inches. By contrast, the 200 fps shot only reaches about 47 feet over the course of 0.32 seconds.